Eighth Texas International Conference on Approximation Theory

College Station Hilton Conference Center

January 8-12, 1995

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Organized by
Charles K. Chui
Larry L. Schumaker
Ewald Quak
N. Sivakumar
Joseph Ward
Conference Secretary: Jan Want

Sponsored by
National Science Foundation
with additional support from
U.S. Army Research Office
and
Texas A&M University

SOCIAL EVENTS

Welcome Party

Sunday, January 8, 7:00 – 8:30 p.m.
Hilton

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Banquet

Tuesday, January 10, 7:45 p.m.
Cash Bar at 7:00 p.m.
Hilton

Banquet: ticket US$25 – vegetarian entree available upon request

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	Sept. 1, 1995	Final Report, Jan. 10, 1995 - Sept. 9, 1995

4. TITLE AND SUBTITLE

International Conference on Approximation Theory and Related Interdisciplinary Topics

5. FUNDING NUMBERS

DAAH04-95-1-0084

DTIC ELECTED S OCT 27 1995 G D

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

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8. PERFORMING ORGANIZATION REPORT NUMBER

RF-8907

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

ARO 33685.1-MA-CF

11. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

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12b. DISTRIBUTION CODE

19951026 034

13. ABSTRACT *(Maximum 200 words)*

The 8th Texas International Conference on Approximation Theory, sponsored by NSF and the U.S. Army Research Office, was hosted by Texas A&M University in College Station, January 8 – 12, 1995. The organizers of this conference were Charles K. Chui, Larry L. Schumaker, Joseph D. Ward, Ewald Quak, and N. Sivakumar. Previous conferences in this series were held in 1973, 1976, 1980, and 1992 in Austin, and 1983, 1986, and 1989 in College Station. The conference was attended by 210 mathematicians from 21 different countries. In addition to a large number of contributed talks, there were three special sessions on interdisciplinary topics of current research interest, arranged by Akram Aldroubi (on Wavelets), Chandrajit Bajaj (on Scientific Visualization), and Joseph Warren (on Computer Geometric Aided Design).

Two volumes of the conference proceedings, carefully edited by C. K. Chui and L. L. Schumaker, will be published by World Scientific Publishing. The central theme of Volume I is the core of approximation theory. It includes such important areas as qualitative approximations, interpolation theory, rational approximations, radial-basis functions, and splines. The second volume focuses on topics related to wavelet analysis, including multiresolution and multi-level approximation, subdivision schemes in CAGD, and applications.

DTIC QUALITY INSPECTED 5

14. SUBJECT TERMS

approximatioon theory; interpolation theory; radial-basis functions; splines; wavelets; scientific visualization, computer geometric aided design; multiresolution and multi-level approximation, subdivsion schemes; applications.

15. NUMBER OF PAGES

2

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

PREFACE

This is the 8th Texas International Conference on Approximation Theory. It is being held in College Station between the 8th and 12th of January, 1995, and is dedicated to our distinguished colleague E. Ward Cheney in honor of his 65th birthday. We extend a very warm welcome to all participants and wish them a pleasant and productive sojourn in 'Aggieland'.

A special feature of this conference is the awarding of the first **Vasil A. Popov Prize** in Approximation Theory to an outstanding young mathematician. This prize will be awarded every three years in this conference series. We thank Ron DeVore for establishing this prize, and for chairing the selection committee which also comprises Charles Chui, Paul Nevai, Allan Pinkus, Pencho Petrushev, and Edward Saff.

One distinctive aspect of this conference is that a special effort has been made to include several interdisciplinary topics of current interest. In this regard, we are most grateful to the following special session chairs: Akram Aldroubi (Sessions 9B and 14B, *Wavelets*), Chandrajit Bajaj (Session 19A, *Scientific Visualization*), and Joseph Warren (Session 24A, *Subdivision Schemes in CAGD*).

In addition to the invited plenary lectures, 177 research presentations have been organized in two parallel sessions. With a few exceptions (due to scheduling constraints and late requests for time changes), these talks have been grouped according to subject areas, with little conflict between the opposing sessions. However, despite our best efforts, these arrangements are bound to fall short of some expectations, and for this we apologize.

As in the case of the previous seven symposia, the National Science Foundation is the major sponsor of this conference. Additional funding from the U.S. Army Research Office† is also acknowledged and appreciated.

The abstracts printed in this program booklet were either reproduced from e-mail messages or typed by the conference secretary, Ms. Jan Want; no stylistic changes were attempted. The booklet itself was prepared by Margaret Chui. It is both our duty and great pleasure to thank Jan and Margaret for their untiring efforts.

The Organizers
January 1995

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† The views, opinions, and/or findings contained in this program booklet are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

CONFERENCE SCHEDULE

	Sunday, January 8	
	Session 1A	**Session 1B**
	Chair: *Hrushikesh Mhaskar*	Chair: *Frank Deutsch*
8:20 a.m.	*Burkhard Lenze, Dortmund, Germany,* On a density question for neural networks	*Jörg Blatter, Rio De Janeiro, Brazil,* On Irving Glicksberg's pseudocompactness paper
8:40 a.m.	*Brad Baxter, London, England,* Compactly supported radial basis functions	*João Prolla, Campinas, Brazil,* Extensions of a theorem of Ky Fan
9:00 a.m.	*Rick Beatson, Christchurch, New Zealand,* Routines for fast computation with radial basis functions	*Bruce Chalmers, Univ. of California, Riverside,* The construction of minimal projections via dual balls
9:20 a.m.	*Gregory Fasshauer, Vanderbilt Univ., Nashville,* Hermite interpolation with radial basis functions on spheres	*Alexis Bacopoulos, U of Montreal, Canada,* Multicriteria in optimization
9:40 a.m.	*Xingping Sun, Southwest Missouri State Univ.,* Strictly positive definite functions on the unit sphere	*S.P. Singh, St. John's, Newfoundland, Canada,* On Fan's best approximation and applications
10:00 a.m.	*Xin Li, Univ. of Nevada, Las Vegas,* Laguerre series method for best Hankel approximation	*Irina Peterburgsky, Suffolk Univ., Mass.,* On the best approximation of abstract integrable functions
	Session 2	
	Chair: *Larry Schumaker*	
11:00 a.m.	*Ward Cheney, Univ. of Texas, Austin,* Approximation using positive definite functions	
	Session 3	
	Chair: *Nira Dyn*	
1:30 p.m.	*Charles Micchelli, IBM, Yorktown Heights,* Title to be announced	
	Session 4A	**Session 4B**
	Chair: *David Handscomb*	Chair: *Manfred Tasche*
2:30 p.m.	*Alain Le Méhauté, Brest, France,* Multivariate Lagrange and Hermite polynomial interpolation	*Bruce Suter, Air Force Institute of Technology, Ohio,* Malvar wavelets on hexagons
2:50 p.m.	*Günther Nürnberger, Mannheim, Germany,* Bivariate spline interpolation and approximation order	*Jianzhong Wang, Sam Houston State Univ., Texas,* Cubic spline-wavelet packets of Sobolev spaces
3:10 p.m.	*Ming-Jun Lai, U of Georgia,* Bivariate splines on triangular regions	*Rudolph Lorentz, St. Augustin, Germany,* Trigonometric polynomial bases of $C[0,1]$
3:30 p.m.	*Dong Hong, U of Texas, Austin,* Optimal triangulations using edge swappings	*George Lorentz, U of Texas, Austin,* Optimal polynomial orthogonal systems

	Sunday, January 8 (Cont'd)	
	Session 5A Chair: *João Prolla*	**Session 5B** Chair: *Kurt Jetter*
4:30 p.m.	*Michael Ganzburg, Courant Institute, New York,* Relations between best approximations	*Kang Zhao, U of Utah,* Approximation in norm and seminorm
4:50 p.m.	*Martin Bartelt, Christopher Newport Univ.,* Haar theory in $C_0(T, R^k)$	*Y.K. Hu, Georgia Southern Univ.,* Convex spline approximation in L_p
5:10 p.m.	*Martina Finzel, Erlangen, Germany,* On best linear approximation in $l_1(n)$	*Peter Köhler, Hildesheim, Germany,* Asymptotic L_q spline approximation and quadrature formulae
5:30 p.m.	*Miguel Marano, Jaén, Spain,* Characterization of best approximants in L_φ spaces	*Kirill Kopotun, Edmonton, Canada,* On k-monotone polynomial and spline approximation
5:50 p.m.	*Oleg Davydov, Ukraine,* Best uniform approximations of periodic functions by convex classes	*Vasant Ubhaya, North Dakota State Univ.,* Data fitting by integer quasiconvex functions
6:10 p.m.	*Lubomir Dechevsky, Sofia, Bulgaria,* Schatten-von Neumann classes versus Besov spaces	*V.P. Sreedharan, Michigan State Univ.,* An algorithm for minimal norm solutions of linear inequalities
6:30 p.m.	*Vassili Demidovitch, Moscow State Univ., Russia,* Error estimates for generalized Chebyshev polynomial interpolations	*Jia-ding Cao, Fudan Univ., China,* Approximation by Boolean sums of linear operators

	Monday, January 9	
	Session 6A	**Session 6B**
	Chair: *Hubert Berens*	Chair: *Johan de Villiers*
8:20 a.m.	*Steven Damelin, Witwatersrand Univ., S Africa,* Approximation theory for Erdös weights	*Debao Chen, Oklahoma State Univ.,* Spline wavelets with prescribed shape
8:40 a.m.	*T.Z. Mthembu, North Qwaqwa Univ., S Africa,* L^∞ Markov-Bernstein inequalities for Erdös weights	*George Donovan, Georgia Tech,* Orthogonal polynomials and construction of scaling functions
9:00 a.m.	*Matthew He, Nova Southeastern Univ., Florida,* Orthogonality of weighted Faber polynomials	*Jeff Marasovich, Vanderbilt Univ.,* Quantizing multiwavelet coefficients
9:20 a.m.	*Elizabeth Kochneff, Eastern Washington Univ.,* On Laguerre polynomials of negative index	*Ilona Weinreich, Aachen, Germany,* Non-stationary MRA and biorthogonal systems
9:40 a.m.	*Bernd Fischer, Lübeck, Germany,* Orthogonal polynomials and solving linear systems of equations	*Stephan Dahlke, Aachen, Germany,* Continuous wavelet transform on tangent bundles
10:00 a.m.	*Pencho Petrushev, Sofia, Bulgaria,* Approximation properties of neural networks	*Doug Hardin, Vanderbilt Univ.,* Orthogonal multiwavelet constructions in 2D
	Session 7	
11:00 a.m.	*Will Light, Leicester Univ., England,* An overview of Cheney's Work	
	Session 8	
	Chair: *Ron DeVore*	
1:30 p.m.	*Vasil A. Popov Prize Winner Lecture*	
	Session 9A	**Wavelets Session I**
	Chair: *Werner Haußmann*	Chair: *Akram Aldroubi*
2:30 p.m.	*Herbert Stahl, Berlin, Germany,* Asymptotic distribution of poles and zeros of best rational approximants to $\|x\|^\alpha$ on $[-1,1]$	*Ingrid Daubechies, Princeton Univ.,* Better dual functions for Gabor lattices
2:50 p.m.	*Marcel de Bruin, Delft, The Netherlands,* Equiconvergence of simultaneous Hermite-Padé interpolants	*Ron DeVore, U of S Carolina,* Nonlinear wavelet approximation
3:10 p.m.	*Roland Freund, AT&T Bell Labs,* Computation of Padé approximations of transfer functions via the Lanczos process	*Wim Sweldens, U of S Carolina,* A new construction of wavelets: the lifting trick
3:30 p.m.	*Kathy Driver, Witwatersrand Univ., S Africa,* Non-diagonal quadratic Hermite-Padé approximation to the exponential function	*Akram Aldroubi, National Institute of Health,* Oblique projections in unitary invariant spaces

	Monday, January 9 (Cont'd)	
	Session 10A	**Session 10B**
	Chair: *Pencho Petrushev*	Chair: *Wim Sweldens*
4:30 p.m.	*V. Belyi, Donetsk, Ukraine,* Approximation of analytic functions by analytic complex planar splines in the quasidisk	*Eberhard Schmitt, Göttingen, Germany,* Approximation on $SO(3)$ and S^3 by splines and wavelets
4:50 p.m.	*Thomas Bloom, Toronto, Canada,* Convergence of Kergin interpolations	*Joe Lakey, Texas A&M Univ.,* Composite wavelet transforms
5:10 p.m.	*Ulrike Maier, Dortmund, Germany,* Evaluation of Kergin interpolants	*Kuei-Fang Chang, Tai-Chung, Taiwan,* A wavelet-like unconditional basis
5:30 p.m.	*Jean-Paul Calvi, Toulouse, France,* A new look at the Fekete-Szegö theorem	*Mohsen Maesumi, Lamar Univ., Texas,* A lower bound for the generalized spectral radius of matrices
5:50 p.m.	*C.H. Lutterodt, Howard Univ., Washington D.C.,* Meromorphic approximation in a Levi pseudoconvex domain	*Manos Papadakis, Athens, Greece,* An equivalence relation between multiresolution analyses of $L^2(\mathbf{R})$
6:10 p.m.	*Zeev Ditzian, Edmonton, Canada,* Best approximation and K-functionals	*Leonardo Traversoni, Iztapalapa, Mexico,* Quaternions on wavelet problems
6:30 p.m.	*Paul Nevai, Ohio State Univ.,* Perturbation of orthogonal polynomials on an arc	*Amos Ron, U of Wisconsin–Madison,* Smooth refinable functions provide good approximation
6:50 p.m.	*Badr saad badr, Tanta, Egypt,* On spline approximations to the solution of a system of nth order ODEs	*Igor Ya. Novikov, Voronezh, Russia,* Non-stationary wavelet bases for Sobolev spaces

	Tuesday, January 10	
	Session 11A	**Session 11B**
	Chair: *Theodore Kilgore*	Chair: *Joachim Stöckler*
8:20 a.m.	*Lyubomir Boyadjiev, Graceland College, Iowa,* Orthogonal polynomial series representations of entire functions	*Frauke Sprengel, Rostock, Germany,* Multivariate periodic interpolation and wavelets
8:40 a.m.	*Eli Levin, Tel Aviv, Israel,* Asymptotic distribution of zeros and poles of rational functions	*Manfred Tasche, Rostock, Germany,* Interpolating wavelets on $[-1,1]$ and the sphere
9:00 a.m.	*Victor Maimeskul, Donetsk, Ukraine,* Rate of uniform rational approximation on arcs	*Jürgen Prestin, Rostock, Germany,* Polynomial bases for spaces of continuous functions
9:20 a.m.	*David Leeming, Victoria, Canada,* Monotone rational approximation	*Norman Weyrich, Air Force Institute of Technology, Ohio,* Algorithms for trigonometric wavelet packets and applications
9:40 a.m.	*Suzanne Thiry, Namur, Belgium,* Rational approximation for design of iterative methods	*Ewald Quak, Texas A&M Univ.,* Trigonometric wavelets for Hermite interpolation
10:00 a.m.	*Ed Saff, Univ. of S Florida,* Rational approximation with varying weights	*Gregory Warhola, Air Force Institute of Technology, Ohio,* De-noising using wavelets and cross validation
	Session 12	
	Chair: *Peter Oswald*	
11:00 a.m.	*Wolfgang Dahmen, Aachen, Germany,* Multiscale analysis, approximation, and interpolation spaces	
	Session 13	
	Chair: *Ed Saff*	
1:30 p.m.	*Doron Lubinsky, Witwatersrand Univ., S Africa,* Weighted approximation and orthogonal polynomials on $\mathbf{R}$	
	Session 14A	**Wavelets Session II**
	Chair: *Katherine Balázs*	Chair: *Akram Aldroubi*
2:30 p.m.	*Hubert Berens, Erlangen, Germany,* Riesz-summability for multivariate inverse Fourier integrals	*Zuhair Nashed, U of Delaware,* Reproducing kernel Hilbert spaces in harmonic analysis
2:50 p.m.	*Yuan Xu, U of Oregon,* Fejér means of multiple Fourier series	*Björn Jawerth, U of S Carolina,* Wavelet smoothing techniques and interactive surface modeling
3:10 p.m.	*Manfred v. Golitschek, Würzburg, Germany,* Approximation by separable functions	*Christian Houdre, Georgia Tech,* Recovery of bandlimited weakly stationary processes
3:30 p.m.	*Werner Haußmann, Duisburg, Germany,* Peano kernel for harmonicity differences	*Christopher Heil, Georgia Tech,* Existence and accuracy for matrix refinement equations

	Tuesday, January 10 (Cont'd)	
	Session 15A	**Session 15B**
	Chair: *Ming-Jun Lai*	Chair: *Bernd Fischer*
4:30 p.m.	*Wu Li, Old Dominion Univ.,* Minimization of multivariate quadratic splines	*Katherine Balázs, Auburn Univ.,* On some constants in simultaneous approximation
4:50 p.m.	*Gabriele Steidl, Darmstadt, Germany,* On multivariate attenuation factors	*Xiang Ming Yu, Southwest Missouri State Univ.,* On copositive polynomial approximation in $L_p[-1,1]$
5:10 p.m.	*David Handscomb, Oxford Univ.,* Error bounds for linear interpolation on triangles	*Detlef Mache, Dortmund, Germany,* Durrmeyer polynomials with Jacobi weights
5:30 p.m.	*Aleksei Shadrin, Novosibirsk, Russia,* Unconditional convergence of multivariate D^m-splines	*Michael Felten, Dortmund, Germany,* Algebraic moduli of smoothness and best approximations
5:50 p.m.	*Vladimir Vasilenko, Novosibirsk, Russia,* Algorithms for multivariate spline approximation	*Antonio da Silva, Rio de Janeiro, Brazil,* Jackson-type theorems on the real line
6:10 p.m.	*Thomas Kunkle, Charleston, S Carolina,* Generalized exponential box splines	*Burkard Polster, Christchurch, New Zealand,* Interpolation and incidence geometry
6:30 p.m.	*Yuesheng Xu, North Dakota State Univ.,* Construction of wavelets on finite domains	*Theodore Kilgore, Auburn Univ.,* Some inequalities for derivatives
6:50 p.m.	*Guanglu Zhang, Dongying, China,* Integral representation of bivariate splines	*Frank Pinter, Leuven, Belgium,* Perturbation of orthogonal polynomials on an arc

	Wednesday, January 11	
	Session 16A	**Session 16B**
	Chair: *Yuan Xu*	Chair: *Jürgen Prestin*
8:20 a.m.	*Knut Petras, Braunschweig, Germany,* Polynomial interpolation and Peano kernels	*Tian-Xiao He, Illinois Wesleyan Univ.,* Spline interpolation and its wavelet analysis
8:40 a.m.	*Ricardo Estrada, Texas A&M Univ.,* Asymptotic approximation by cardinal series	*Tom Hogan, U of Wisconsin, Madison,* Shifts of multivariate refinable functions
9:00 a.m.	*Armin Iske, Göttingen, Germany,* Title to be announced	*Suresh Lodha, U of California, Santa Cruz,* Bézier wavelets
9:20 a.m.	*Thomas Sauer, Erlangen, Germany,* Newton method for polynomial interpolation	*Gerlind Plonka, Rostock, Germany,* Multiwavelets with short support
9:40 a.m.	*Jeremy Levesley, Leicester, England,* Generalised *sk*-spline interpolants on compact abelian groups	*Gina Puccio, Messina, Italy,* Recognition of biological shapes by wavelet analysis
10:00 a.m.	*Johnnie Baker, Kent State Univ., Ohio,* Sorting algorithm for reconfigurable meshes and networks	*Mei Kobayashi, Tokyo, Japan,* Wavelet analysis of speech signals
	Session 17	
	Chair: *George Lorentz*	
11:00 a.m.	*Gilles Pisier, Texas A&M Univ. and Univ. of Paris VI,* Real and complex interpolation of operator spaces	
	Session 18	
	Chair: *Björn Jawerth*	
1:30 p.m.	*Ingrid Daubechies, Princeton Univ.,* Determination of regularity of refinable functions	
	Scientific Visualization Session	**Session 19B**
	Chair: *Chandrajit Bajaj*	Chair: *Christopher Heil*
2:30 p.m.	*B.K. Natarajan, Hewlett Packard,* Sparse approximate multiquadric interpolation	*Kurt Jetter, Duisburg, Germany,* de Boor's stability result and symmetric preconditioning
2:50 p.m.	*Tom Foley, Arizona State Univ., Tempe,* Plane preserving volume deformations using scattered landmark points	*Joachim Stöckler, Duisburg, Germany,* Wavelet analysis of scattered data
3:10 p.m.	*Marian Neamtu, Vanderbilt Univ.,* Spherical splines	*Fran Narcowich, Texas A&M Univ.,* Nonstationary spherical wavelets for scattered data
3:30 p.m.	*Chandrajit Bajaj, Purdue Univ.,* Modeling and visualizing C^2 scattered function data	*Joe Ward, Texas A&M Univ.,* Wavelets associated with periodic basis functions

	Wednesday, January 11 (Cont'd)	
	Session 20A	**Session 20B**
	Chair: *Xiang Ming Yu*	Chair: *Ewald Quak*
4:30 p.m.	*Dany Leviatan, Tel Aviv, Israel,* Convex polynomial approximation in the L_p norm	*Philip Smith, WindWard Technologies,* Approximation on compact sets by radial basis functions
4:50 p.m.	*George Anastassiou, U of Memphis,* Differentiated shift-invariant integral operators	*Hein Hundal, Penn State Univ.,* Generalizations of Dykstra's algorithm
5:10 p.m.	*Mi Zhou, U of Memphis,* Asymptotic expansions for certain operator semigroups	*Junjiang Lei, Southern Illinois Univ.,* Integral operators in shift-invariant spaces
5:30 p.m.	*Elisabetta Santi, Rome, Italy,* Numerical solution of singular integral equations of Cauchy type	*Gisela Pöplau, Rostock, Germany,* Fast Poisson solvers in shift-invariant spaces
5:50 p.m.	*Kurt Riedel, Courant Institute, New York,* Piecewise convex function estimation and model selection	*Shayne Waldron, U of Wisconsin–Madison,* Error represention in multivariate polynomial interpolation
6:10 p.m.	*Alex Sidorenko, Courant Institute, New York,* Adaptive kernel estimation of spectral density	*Tianping Chen, Fudan Univ., China,* Radial basis functions and applications to neural networks
6:30 p.m.	*Lefan Zhong, Beijing, China,* Interpolation bases, the Marcinkiewicz-Zygmund inequality and A_p weights	*Vladimir Andrievskii, Eichstätt, Germany,* Uniform polynomial approximation on a quasidisk
6:50 p.m.	*M.A. Bokhari, Dhahran, Saudi Arabia,* L_2 and L_∞-approximation with interpolatory constraints	*D.V. Pai, Bombay, India,* Continuity of the prox and restricted center maps

	Thursday, January 12	
	Session 21A Chair: *N. Sivakumar*	**Session 21B** Chair: *Laura Montefusco*
8:20 a.m.	*Guanrong Chen, U of Houston,* Algorithms for matrix-valued Nevanlinna-Pick interpolation	*Jian-ao Lian, Prairie View A&M Univ., Texas,* Characterization of polynomial reproduction for multiscaling functions
8:40 a.m.	*X.-L. Liu, Vanderbilt Univ.,* Bézier triangle patches on sphere-like surfaces	*Alexander Petukhov, St. Petersburg, Russia,* Nonorthogonal compactly supported wavelets
9:00 a.m.	*Sonya Stanley, Vanderbilt Univ.,* Circular Bernstein-Bézier curves and degree raising	*Bl. Sendov, Sofia, Bulgaria,* Compression through rotations
9:20 a.m.	*I.J. Anderson, Huddersfield, England,* Matrix methods for approximation of multivariate data	*C. Guerrini, Bologna, Italy,* Parallel wavelet-Galerkin methods using wavelet packets
9:40 a.m.	*Ranjan Muttiah, Texas A&M Univ.,* Climate surfaces using thin plate splines	*Gerhard Opfer, Hamburg, Germany,* Cardinality of orthogonal wavelet multiplier matrices
10:00 a.m.	*G.A. Watson, Dundee, Scotland,* An approximation problem in measurement analysis	*Karsten Urban, Aachen, Germany,* Divergence-free wavelet bases
	Session 22 Chair: *Tom Lyche*	
11:00 a.m.	*Rong-Qing Jia, Edmonton, Canada,* Refinable shift-invariant spaces: from splines to wavelets	
	Session 23 Chair: *Will Light*	
1:30 p.m.	*Robert Schaback, Göttingen, Germany,* Multivariate interpolation and approximation	
	Subdivision Scheme Session Chair: *Joe Warren*	**Session 24B** Chair: *Joe Lakey*
2:30 p.m.	*Nira Dyn, Tel Aviv, Israel,* Nonstationary subdivision schemes and multiresolution analysis	*Peter Oswald, Texas A&M Univ.,* Piecewise linear prewavelets with small support
2:50 p.m.	*Ulrich Reif, Stuttgart, Germany,* Estimation of curvature continuous subdivision surfaces	*Johan de Villiers, Stellenbosch, S Africa,* Construction of fundamental interpolants and wavelets
3:10 p.m.	*Leif Kobbelt, Karlsruhe, Germany,* A variational approach to subdivision	*Laura Montefusco, Bologna, Italy,* Adapted wavelet packets for parallelism in image restoration
3:30 p.m.	*Joe Warren, Rice Univ., Texas,* Smooth functional subdivision methods for irregular triangulations	*Sônia Gomes, Campinas, Brasil,* Numerical algorithms for periodic biorthogonal wavelets

	Thursday, January 12 (Cont'd)	
	Session 25A Chair: *Alistair Watson*	**Session 25B** Chair: *Joe Ward*
4:30 p.m.	*Francois Dubeau, Sherbrooke, Québec,* Interlacing properties of roots of Euler-Frobenius polynomials	*M. Skopina, St. Petersburg, Russia,* Pointwise convergence of periodized wavelet series
4:50 p.m.	*J. Savoie, Richelain, Québec,* Optimal error bounds for spline interpolations	*Shuzhan Xu, Edmonton, Canada,* Computing continuous wavelet transforms by subdivision
5:10 p.m.	*J. Koch, Stuttgart, Germany,* Geometric Hermite interpolation	*Valery A. Zheludev, St. Petersburg, Russia,* On signal processing using periodic spline wavelets
5:30 p.m.	*Barbara Bertram, Michigan Technological Univ.,* Divergent Neumann sequences for Fredholm integral equations	*Chun Li, Texas A&M Univ.,* Multivariate interpolating wavelets and their duals
5:50 p.m.	*V.V. Zhuk, St. Petersburg, Russia,* Approximation of continuous functions of bounded variations	*Jianrong Wang, HARC, Texas,* n-multiple wavelets with n-multiple vanishing moments
6:10 p.m.	*Shusheng Xu, Jiangsu, China,* Characterization of best L_p approximation with restricted derivatives	*S.Ya. Novikov, Samara, Russia,* Canonical injection operators for rearrangement invariant spaces
6:30 p.m.	*Avraham Melkman, Beersheba, Israel,* An efficient variation on the Oslo algorithm	*Vadim Iv. Filippov, Saratov, Russia,* Classification of function systems in L_p and wavelets
6:50 p.m.	*Xiaolei Dong, Harbin, China,* Best approximation on function spaces	

ABSTRACTS

Oblique Projections in Unitary Invariant Spaces

Akram Aldroubi
National Institutes of Health, Biomedical Engineering
and Instrumentation Program
Bethesda, Maryland

Let $\mathcal{H}$ be a Hilbert space, and let O be a unitary operator on $\mathcal{H}$. Using the operator O, and a set of vectors $\{\varphi^i\}_{i=1,...N}$ in $\mathcal{H}$ we construct an *O-invariant subspace* $U \subset \mathcal{H}$:

$$U\left(O, \{\varphi^i\}_{i=1,...N}\right) = \left\{ \sum_{i=1}^{N} \sum_{k \in \mathbf{Z}} c^i(k) O^k \varphi^i; \quad c^i \in l_2,\ i = 1, 2, ..., N \right\}. \tag{1}$$

We give the necessary and sufficient conditions for U to be a well-defined subspace of $\mathcal{H}$ with $\left\{O^k \varphi^i\right\}_{k \in \mathbf{Z},\ i=1,2...,N}$ as its Riesz basis. We then consider the oblique projection $P_{U \perp V}$ on the space $U(O, \{\varphi^i\}_{i=1,...N})$ in a direction orthogonal to $V(R, \{\tau^i\}_{i=1,...N})$. We give necessary and sufficient conditions on $O, R, \{\varphi^i\}_{i=1,...N}, \{\tau^i\}_{i=1,...N}$ for $P_{U \perp V}$ to be well defined. We then use the results to construct the theory of biorthogonal-multi-wavelets which is the generalization of the Goodman, Lee, and Tang theory of wavelets in wandering subspaces.

Differentiated Shift-Invariant Integral Operators, Univariate Case

George A. Anastassiou
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In a recent paper the author, along with H. Gonska, introduced some wavelet type integral operators over the whole real line and studied their properties such as shift-invariance, global smoothness preservation, convergence to the unit, and preservation of probability distribution functions. These operators are very general and they are introduced through a convolution-like iteration of another general operator with a scaling type function. In this paper the author provides sufficient conditions, so that the *derivatives of the above operators* enjoy the same nice properties as their originals. A sufficient condition is also given so that the "global smoothness preservation" related inequality becomes sharp. At the end several applications are given, where the derivatives of the very general specialized operators are shown to fulfill all the above properties. In particular it is shown that they preserve continuous probability density functions.

Efficient Matrix Methods for the True Least-Squares Approximation of Structured Multivariate Data

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It is well-known that very efficient algorithms may be derived for approximating data whose abscissae lie on a mesh, based on the idea of separating the variables. The present authors and co-workers have extended this separation-of-variables approach in a number of papers and developed algorithms for other data sets, which have structure but to a lesser extent, such as data on or near to a set of lines, data on a family of curves, and data which are uniformly scattered. Generally these algorithms do not produce "true" least-squares approximations but rather "near-best" approximations. However, an alternative approach to such problems is to set up the complete overdetermined system of observation equations, without separating variables, and then to exploit the special structures that exist in the matrix, in order to perform a rapid QR decomposition. In this paper we illustrate this approach for data on a family of lines, showing that an efficient method can be devised which yields the true least-squares solution. The method is valid for a wide variety of basis functions, but can be further refined in the case of spline functions, to take account of local support features.

Uniform Polynomial Approximation on a Quasidisk

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This report is connected with the study of the values $E_n(f,\bar{G})$, $n = 0,1,2\ldots$, of best uniform polynomial approximations of a function f analytic in a bounded Jordan domain $G \subset \mathbf{C}$ and continuous on its closure $\bar{G}$. The rate of decrease of $E_n(f,\bar{G})$ as $n \to \infty$, the geometric structure of the boundary ∂G of G, and the smoothness of the function f near the boundary interact in a complicated way. The main subject of our talk is the consideration of the following two problems. Let $\mu(\delta), \delta > 0$, be a so-called normal majorant (for example, $\mu(\delta) = \delta^c$, $c = \text{const} > 0$).

PROBLEM A. Describe all functions f satisfying

$$E_n(f,\bar{G}) = 0(\mu(1/n)) \quad \text{as} \quad n \to \infty.$$

PROBLEM B. Describe all functions f for which

$$E_n(f,\bar{G}) \sim \mu(1/n), \quad n = 1,2,\ldots,$$

where the symbol $g \sim \varphi$ means that

$$1/c \leq g\varphi \leq c$$

holds for some constant $c > 0$.

We give the solution of Problems A and B in the case when G is an arbitrary quasidisk and apply then these results to the study of a problem of Turan concerning the correlation between polynomial and rational approximations on the unit disk.

Multicriteria Optimization

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We define a convenient framework of vector norms and give some results in multicriteria optimization. Theorems of existence, characterizations, uniqueness and computation of best approximations are given here in three mutually related contexts of optimality. Parallel computation and numerical schemata of optimization are discussed.

Error Of An Arbitrary Order For The Approximate Solution Of System Of nth Order Differential Equations With Spline Functions

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In this paper we present a method for approximating the solution of a system of nonlinear nth order differential equations $y^{(n)}(x) = f_1(x, y, z), z^{(n)}(x) = f_2(x, y, z)$ with $y^{(j)}(x_0) = y_0^{(j)}, z^{(j)}(x_0) = z_0^{(j)}, \quad j = 0, 1, ..., n-1$. We use spline functions for finding the approximate solution. It is a one-step method where the error in $y^{(n)}, z^{(n)}$ is of order $O\left(h^{nm+r+\alpha}\right)$, provided that $f_1, f_2 \in C^r\left([0,1] \times R^2\right)$ and the modulus of continuity of $y^{(n)}$, $z^{(n)}$ is $O\left(h^{\alpha}\right)$,where $r \in N, i = 0, 1, ..., n+r$, $0 < \alpha < 1$ and $m \geq 1$ is an arbitrary positive integer which equals the number of iterations used in computing the spline functions defined in the method. It is also shown that the method is stable.

Modeling and Visualizing C^2 Scattered function Data on Curved Surfaces

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We shall present algorithms for constructing a C^1 cubic or a C^2 quintic piecewise trivariate Bernstein-Bézier polynomial approximation F of scattered function data defined

on an arbitrary smooth domain surface D in three dimensional real space ($\mathbb{R}^3$). The smooth polynomial pieces or finite elements of F are defined on a three dimensional triangulation called the simplicial hull surrounding the domain surface D. We shall also present techniques for visualizing the isocontours of the function F. The smooth polynomial approximation F allows one to additionally manipulate the local geometry of the modeled function and supports applications in data visualization, navigation and querying.

A Constant Time Sorting Algorithm for the 3-D Reconfigurable Mesh and the Reconfigurable Network

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Sorting techniques have numerous applications. Current real number and integer sorting techniques for the reconfigurable mesh operate in constant time using a reconfigurable mesh of size $n \times n$ to sort N numbers. This paper presents a constant time algorithm to sort n items on a reconfigurable network with $O(\sqrt{n} \times \sqrt{n} \times \sqrt{n})$ switches and $O(\sqrt{n} \times \sqrt{n})$ processors. Also, new constant time selection and compression algorithms are obtained. All results may also be implemented on the 3-D reconfigurable mesh.

On Some Constants in Simultaneous Approximation

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Pointwise estimates for the error which is feasible in simultaneous approximation of a function and its derivatives by an algebraic polynomial were originally pursued from theoretical motivations, which did not immediately require the estimation of the constants in such results. However, recent numerical experimentation with traditional techniques of approximation such as Lagrange interpolation, slightly modified by additional interpolation of derivatives at ± 1, shows that rapid convergence of an approximating polynomial to a function and of some derivatives to the derivatives of the function is often easy to achieve. The new techniques are theoretically based upon older results about feasibility, contained in work of Trigub, Gopengauz, Telyakovskii, and others, giving new relevance to the investigation of constants in these older results. We begin this investigation here. Helpful in obtaining estimates for some of the constants is a new identity for the derivative of a trigonometric polynomial, based on a well known identity of M. Riesz. One of our results is a new proof of a theorem of Gopengauz which reduces the problem of estimating the constant there to the question of estimating the constant in a simpler theorem of Trigub used in the proof.

Haar Theory in $C_0(T, R^k)$

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In 1956 S. B. Stechkin and S. I. Zukhovickii characterized those finite dimensional subspaces of $C_0(T, R^k)$ from which best uniform approximations are unique. Using these subspaces we extend the Haar Theory involving strong unicity and Lipschitz continuity from $C(X)$ to $C_0(T, R^k)$.

Compactly Supported Radial Basis Functions

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A radial basis function approximation has the form

$$s(x) = \sum_{k=1}^{n} a_k \varphi(x - b_k), \qquad x \in \mathcal{R}^d, \tag{1}$$

where $\varphi: \mathcal{R}^d \to \mathcal{R}$ is some given (usually radially symmetric) function, $(a_k)_1^n$ are real coefficients, and the centres $(b_k)_1^n$ are points in $\mathcal{R}^d$. We investigate some properties of φ when it has the distributional Fourier transform $\widehat{\varphi}(\xi) = c\|\xi\|^{-2m}$, where c is a real number and m is a positive integer. The class of such functions includes the Euclidean norm $\varphi(x) = \|x\|$ when the dimension d is odd and the thin plate spline $\varphi(x) = \|x\|^2 \log \|x\|$ when $d = 2$. Our key observation is that it is easy to construct a signed measure μ on $\mathcal{R}^d$ for which the function $\hat{\mu}(\xi)\widehat{\varphi}(\xi)$ is an entire function of exponential type. Hence the Paley-Wiener theorem allows us to conclude that the convolution $\mu * \varphi$ is a compactly supported function. One consequence of this result is a local stability estimate in the uniform norm for the function s defined by (1): If $\|b_j - b_k\| > 1$ for $j \neq k$, then

$$|a_j| \leq \max\{|s(x)| : \|x - b_j\| \leq 1\}/\Delta_{2m-1}, \qquad j = 1, \ldots, n,$$

where Δ_{2m-1} denotes the distance in $C[-1, 1]$ of the univariate function $\{\varphi(tu) : t \in \mathcal{R}\}$ from the space of polynomials of degree $2m-1$, u being any unit vector in $\mathcal{R}^d$. Another is a norm estimation technique generalizing earlier research. Further, we find that it is possible to choose the coefficients $(a_k)_{k=1}^n$ and the centres $(b_k)_{k=1}^n$ such that the *spherical average* As of s is compactly supported, where the spherical average of a continuous function $f : \mathcal{R}^d \to \mathcal{R}$ is defined by the equation

$$Af(x) = \int_{O_d} f(Ux)\, d\sigma(U), \qquad x \in \mathcal{R}^d. \tag{2}$$

Here O_d denotes the group of real $d \times d$ orthogonal matrices and σ_d denotes the group invariant probability measure on O_d.

THIN: A Collection of Routines for Fast Computation With Radial Basis Functions

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If radial basis functions are used in a direct way then the demands in time and storage can be excessive. There are several ways to reduce these demands. This talk concerns a collection of routines implementing some of these fast algorithms. Initially routines for fast evaluation, and fast solution of the interpolation equations will be included. It is hoped to add further routines, such as fast GCV, at a later date.

On a Problem of Approximation to the Analytic Function by Analytic Complex Planar Splines in the Quasidisk

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In fact, the problem of recovering of analytic function in a quasidisk by complex planar splines is a problem of spline approximation. Here we consider the approximation of functions which are analytic in the bounded quasidisk G and continuous over its closure $\overline{G}$. Let $S_{\Delta_N}(f;z)$ be an analytic complex spline defined on triangular or rectangular grids,

$$S_{\Delta_N}(f;z) := -\frac{1}{\pi}\int\limits_G \frac{s_{\Delta_N}(\zeta)}{(y(\zeta)-z)^2} y_{\bar{\zeta}}\, d\sigma_\zeta, \qquad (z \in G), \tag{1}$$

where $s_{\Delta_N}(\zeta)$ is the complex planar spline of G. Opfer—M. L. Puri, and $y(z)$ is the quasiconformal reflection of the complex plane with respect to $L = \partial G$. Our main result is:

$$\|f(z) - S_{\Delta_N}(f;z)\|_{C(\bar{G})} \le M\omega(f;h_N), \tag{2}$$

where $\omega(f;t)$ is the modulus of continuity of f, h_N the mesh width and M depends on $\bar{G}$ and f . The estimation (2) was known earlier for the case of the quasismooth boundary $\partial\bar{G}$ only.

Some other results concerning problems of approximation by analytic splines are considered.

On Riesz-summability for Multivariate Inverse Fourier Integrals

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This is joint work with Yuan Xu. Motivated by Yuan's investigations of multivariate orthogonal polynomials, we were led to study, what we call, ℓ-1 summability of inverse Fourier integrals and Fourier series.
Let $f \in L^1(\mathbf{R}^d)$, $d \in \mathbf{N}$, and let $\widehat{f}$ be its Fourier integral. We denote by

$$D^{(1)}_{R;d}(x) = \int_{|v|_1 \le R} e^{iv\cdot x}\, dv, \qquad x \in \mathbf{R}^d \quad \text{and} \quad R > 0$$

the ℓ-1 Dirichlet-kernel and by

$$S^{(1)}_{R;d}(f;x) = \int_{|v|_1 \le R} e^{iv\cdot x}\, \widehat{f}(v)\, dv = f \star D_{R;d}(1)(x)$$

the inverse Fourier integral of f – note that we are integrating over the ℓ_1-ball in $\mathbf{R}^d$ centered at the origin with radius $R > 0$. As a central result we proved: Let $\delta > 0$. If

$$S^{(1)}_{R;\delta;d}(f;x) = \int_{|v|_1 \le R} \left(1 - \frac{|v|_1}{R}\right)^{\delta} e^{iv\cdot x}\, \widehat{f}(v)\, dv, \qquad x \in \mathbf{R}^d \quad \text{and} \quad R > 0;$$

denote the Riesz (R,δ) means of the inverse Fourier integral of f, we have

(1) the means define an approximate identity on $L^1(\mathbf{R}^d)$ (and consequently on $C_o(\mathbf{R}^d)$ and $L^p(\mathbf{R}^d)$, $1 < p < \infty$)); i. e.,

$$\forall f \in L^p(\mathbf{R}^d) \qquad \lim_{R\to\infty} S^{(1)}_{R;\delta;d}(f;\cdot) = f \qquad \text{in norm.}$$

(2)

$$\forall f \in L^p(\mathbf{R}^d) \qquad |S^{(1)}_{R;\delta;d}(f;| \le \mathrm{const}_{d;\delta}\, M(f;x) \qquad a.\ e.,$$

where $M(f;\cdot)$ is the maximal function of f; in particular,

$$\forall f \in L^p(\mathbf{R}^d) \qquad \lim_{R\to\infty} S^{(1)}_{R;\delta;d}(f;x) = f(x) \qquad \text{a. e.}$$

This is in sharp contrast to the well-known Bochner–Riesz means which have a critical index (of convergence), namely, $\delta > (d-1)/2$.

The proof depends strongly on a closed form of the Dirichlet kernel as a divided difference of a function on $\mathbf{R}_+$ with knots being the square of the coefficients of $x = (x_1, \ldots, x_d) \in \mathbf{R}^d$. There are analogous results for Fourier series.

Numerical Transformations of Divergent Neumann Sequences for Fredholm Integral Equations of the Second Kind

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We consider the numerical solution of second kind Fredholm integral equations of the form

$$f(x) = g(x) + \int_a^b k(x,t)f(t), \qquad a < x < b,$$

where the underlying integral operator admits discrete spectra but has sup norm greater than (or equal to) one. In general, the application of successive approximations (Neumann series) is precluded because it is divergent. Abbreviating (1) as $f = g + Lf$, we transform the Neumann sequence, $f_n = \sum_{j=0}^{n-1} L^j g$, using Aitken's method applied pointwise. Somewhat surprisingly, we find in several examples that the transformed sequence converges rapidly to the solution f. Since Aitken's method does not usually sum divergent sequences, the numerical results imply that Neumann sequences have a simple structure. Indeed, a brief formal analysis shows that

$$f_n \simeq f + \sum_{i \geq 1} A_i \lambda_i{}^n,$$

where λ_i are the eigenvalues of L and A_i depends on the independent variable x, but not on n.

Computations were done for the following kernels on the interval [0,1]: $exp(\alpha(x-t)^2)$ (M. Kac), $(1-xt)^{-\alpha}$ (M. L. Glasser), $exp(\alpha xt)$ (H. Brakhage), and $|x-t|^{-\alpha}$ (weakly singular). By varying α, the number of eigenvalues larger than 1 can be controlled. In each case, when α was chosen to make the Neumann sequence diverge, one or more successive Aitken transforms effectively summed it.

A Note on Irving Glicksberg's Pseudocompactness Paper

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Dedicated to the Memory of Galen L. Seever

Let X be *any* non-void topological space and denote by $C(X)$ the set of all real-valued continuous functions on X and by $C^*(X)$ the subset of all bounded real-valued continuous

functions on X. Irving Glicksberg [1], [2] solved the question of when a particular non-negative linear functional I on $C(X)$ can be represented by a Baire-measure μ on X in the sense that

$$I(f) = \int_X f \, d\mu \quad \forall f \in C(X)$$

by proving a pioneering version of what is known today as the Daniell-Stone Theorem and which gives a necessary and sufficient condition on I in order to have such representation. He then went on to investigate the question of when *all* non-negative linear functionals on $C(X)$ have such representation and found that a necessary and sufficient condition for this is that X be pseudo-compact, which is to say that $C(X) = C^*(X)$. Finally, Glicksberg established, with rather complicated proofs, that pseudo-compactness of X is also equivalent to the validity of various other classical theorems on $C(X)$, including Dini's and Ascoli's.

What we do here is

1. Show that pseudo-compactness of X is equivalent to the validity of three more classical theorems on $C(X)$, among these, surprisingly, Arzelá's Theorem, apparently much stronger than Dini's; and
2. give completely elementary proofs of both Glicksberg's results and our own, of course.

References

[1] Irving Glicksberg, The representation of functionals by integrals, *Duke Math. J.* **19** (1952), 253–261.

[2] Irving Glicksberg, Stone-Çech compactifications of products, *Trans. Amer. Math. Soc.* **90**, 369–382.

Convergence Problem for Kergin Interpolation II

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Let E, F, G be three compact sets in C^n. We say that (E, R, G) holds if for any choice of an interpolating array in F and of an analytic function f on G, the corresponding Kergin interpolation polynomials of f exists and converge to f on E. Given G, regular $C-$ convex and E (resp) we construct optimally F (resp E) so that (E, F, G) holds.

On L_2 and L_∞-Approximations Subject to Interpolatory Constraints

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Given n distinct points $x_1, x_2, \ldots, x_n \in [a,b]$, a function $f \in C^r[a,b]$ and an integer $m \geq (r+1)n - 1$, we consider the problem

$$\min\{\|f - h\| : h \in \pi_m, h^{(j)}(x_i) = f^{(j)}(x_i), i = 1, 2, \ldots, n, j = 0, 1, \ldots, r\} \qquad (*)$$

where $\|\cdot\|$ denotes either L_2-norm or L_∞-norm. Using a technique of J. L. Walsh, the problem $(*)$ is modified to an unconstrained minimization problem. We note that the solution of $(*)$ possesses several similarities to the respective classical cases of L_2 and L_∞ approximation problems.

Series Representations of Entire Functions by Laguerre and Hermite Polynomials

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Eigenfunction expansions are sometimes useful in studying the properties of the functions to which they converge. Here we consider entire functions and their expansions in terms of Laguerre and Hermite series. The class of entire functions which have expansions in terms of Hermite series is described using the generating function for Hermite polynomials as a basic tool. The discussion about Laguerre representations of entire functions involve the classical Hankel's integral transformation. Another approach to this problem, based on equiconvergence theorems for series in Laguerre polynomials, is also considered.

Equiconvergence of Some Simultaneous Hermite-Padé Interpolants

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The object is to extend some recent results of E. B. Saff, A. Sharma and R. S. Varga (R.A.I.R.O. Anal. Numér. 15 (1981), no. 4, 371–390) and M. P. Stojanova (Constr. Approx. 4 (1988), 435–445) on equiconvergence of Hermite-Padé interpolation of meromorphic functions to the case of simultaneous interpolation.

Let $d, \nu_1, \nu_2, \ldots, \nu_d$ be natural numbers and let $\vec{F} = (F_1, \ldots, F_d)$ be d functions mermorphic in the discs D_{ρ_i} $(1 < \rho_1 \leq \rho_2 \ldots \leq \rho_d,\ \rho_j$ real) respectively, given by

$$F_i(z) := \frac{f_i(z)}{B_i(z)},\ f_i(z) := \sum_{k=0}^{\infty} a_{i,k} z^k,\ \limsup_{k\to\infty} |a_{i,k}| \leq \frac{1}{\rho_i}\ (i = 1, \ldots, d),$$

where

$$B_i(z) := \prod_{j=1}^{\mu_i} (z - z_{i,j})^{\lambda_{i,j}},\ \sum_{j=1}^{\mu_i} \lambda_{i,j} = \nu_i.$$

Here it is assumed that the poles of the functions are in disjoint sets (i.e. $z_{i,j} \neq z_{k,l}$ when $i \neq k$ and all j, l). Let ℓ be a positive integer and put $n = \sigma + 1$, where $\sigma = \nu_1 + \nu_2 + \cdots + \nu_d$, finally $r \geq 1$ is a natural number. The Hermite-interpolant to the Taylor-sections $\sum_{k=0}^{n\ell r-1} a_{i,k} z^k$ on the zeros of $(z^n - 1)^r$ will be denoted by $\tilde{f}_{i,\ell}(z)$ (note that the case $\ell = \infty$ is well defined).

The interpolation problem with multiple nodes is now formulated for $1 \leq \ell \leq \infty$ as

find d rational functions $U_i^\ell(z)/B^\ell(z)$, $deg\, U_i^\ell(z) \leq nr - \nu_i - 1$ $(1 \leq i \leq d)$, $deg\, B^\ell(z) \leq n - \nu_0 - 1$, that interpolate the d rationals $\tilde{f}_{i,\ell}(z)/B^\ell(z)$ $(1 \leq i \leq d)$ on the zeros of $(z^n - 1)^r$; moreover the coefficient of $z^{n-\nu_0-1}$ in B^ℓ will be 1: $B^\ell(z) = \sum_{k=0}^{n-\nu_0-1} \gamma_k^\ell z^k$, $\gamma_{n-\nu_0-1}^\ell = 1$.

The main result is as follows.

A. For n sufficiently large, the interpolation problem stated above, has a unique solution that moreover satisfies

$$\lim_{n\to\infty} \gamma_k^\ell = \zeta_k,\ \text{with} \sum_{k=0}^{n-\nu_0-1} \zeta_k z^k = \prod_{i=1}^{d} B_i(z);\ 1 \leq \ell \leq \infty.$$

B. Let $\mathcal{H}$ be a compact subset of $|z| < \tau$, $\tau > 0$, that omits the singularities of the functions F_i $(1 \leq i \leq d)$, then

$$\limsup_{n\to\infty} \{\max_{z\in\mathcal{H}} \left| \frac{U_i^\infty(z)}{B^\infty(z)} - \frac{U_i^\ell(z)}{B^\ell(z)} \right|\}^{1/n} \leq c(i, \tau),$$

where $c(i,\tau) = R^{(\ell-1)r+1}(\tau/\rho_i)^r$ for $\tau \geq \rho_i$ and $c(i,\tau) = R^{(\ell-1)r+1}$ for $\tau < \rho_i$ with $R = \max_{1\leq i\leq d} 1/\rho_i$.

C. Specifically we have for $|z| < \rho_i R^{-(\ell-1+1/r)}$:

$$\lim_{n\to\infty} \frac{U_i^\infty(z)}{B^\infty(z)} - \frac{U_i^\ell(z)}{B^\ell(z)} = 0,$$

uniformly and geometrically in compact subsets of $\mathcal{H}$ omitting the singularities.

A New Look at the Fekete-Szegö Theorem

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Let E be a compact point set in $\mathbf{C}$, we assume that its logarithmic capacity is greater than 1 ($cap(E) > 1$). Then a classical [1955] theorem of Fekete and Szegö states that there are only finitely many *monic* polynomials $p(X) \in \mathbf{Z}[i][X]$ irreducible over $\mathbf{Q}(i)$ and having all their roots in E, i.e., $p^{-1}(0) \subset E$.

Thus if $K(E)$ denotes the set of all the roots of these polynomials, we have $cardK(E) < \infty$. However, Fekete and Szegö said nothing more on the size of $K(E)$. In this note, we are interested in finding *specific estimates* for *card* $(K(E))$ under some geometric hypothesis on E. To do so, following the original idea of Fekete, we have to find polynomials $q \in \mathbf{Z}[i][X]$ with "small" degree and such that $\|q\|_E > 1$. (The original method does not seem to provide bounds for the degree.)

We propose a solution which makes use of classical Carleman's inequalities for orthogonal polynomials with respect to the plane Lebesgue measure (on a compact set).

Here $p(X) \in \mathbf{Z}[i][X]$ means that all the coefficients of p belong to $\mathbf{Z}[i] = \{a+ib, a, b \in \mathbf{Z}\}$.

Approximation by Boolean Sums of Linear Pičugov–Lehnhoff Operators: Teljakovskiĭ's Type Estimates and Constants of Approximation

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We estimate constant and find best asymptotic constant in Teljakovskiĭ's Theorem concerning approximation by algebraic polynomials. We research approximation by Boolean sums of linear Pičugov–Lehnhoff operators.

The Construction of Minimal Projections via Dual Balls

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A minimal norm projection $P = \sum_1^n u_i \otimes v_i$ from a Banach space X onto $V = [v_1, \ldots, v_n]$ is obtained as follows:

$$\|a_1, \ldots, a_n\| := \|a_1 v_1 + \ldots + a_n v_n\|_X$$

defines a norm in n-space with ball B.

$u_1, \ldots, u_n$ which make P minimal are determined from the requirement tha

$$\|b_1, \ldots, b_n\|_* := \|b_1 u_1 + \ldots + b_n u_n\|_{X^*}$$

defines a norm in n-space with ball B^*, the dual ball to B.

A Wavelet-like Unconditional Basis

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We construct a bounded and Besselian basis in the space $L^p(\mathbf{R})$ for $1 < p \leq 2$, and we construct a bounded and Hilbertian basis in $L^p(\mathbf{R})$ for $2 \leq p < \infty$. These bases depend on an orthonormal wavelet in $L^2(\mathbf{R})$. Under the Wiener condition, a wavelet and its bi-orthogonal wavelet have the same rate of decay at infinity. If a pre-wavelet has a bi-orthogonal pair of Riesz bases, then the pre-wavelet provides an unconditional basis for all $L^p(\mathbf{R})(1 < p < \infty)$.

Spline Wavelets with Prescribed Shape

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In some previous work, we studied the general structure of cardinal spline wavelets, and drew graphs of some of them, to show the variety of shapes included in these families. In this paper we prove that we can construct spline wavelets with virtually any prescribed shape. As a consequence, we can prove that for any function f satisfying certain mild conditions, the set $\{2^{j/2} f(2^j \cdot -k) : j, k \in \mathbf{Z}\}$, (which may not be a wavelet basis or wavelet frame) is dense in $L^2(\mathbf{R})$.

Fast Algorithms for Matrix-Valued Nevanlinna-Pick Interpolation

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In this talk, we introduce several fast algorithms for both scalar-valued and matrix-valued Nevanlinna-Pick interpolation. Given n distinct points z_i in the unit disk $|z| < 1$

and n (m by m) complex matrices W_k satisfying the Pick condition, the fast Nevanlinna-Pick interpolation algorithms developed by the author require only $0(m^3 \log n)$ complex arithmetic operations to evaluate the matrix-valued interpolatory rational function at any particular value of z, in contrast to the naive Nevanlinna algorithms which require $0(m^3 n)$ arithmetic operations, where they both use $0(m^2 n)$ processors.

Denseness of Radial-Basis Functions in $L^2(R^n)$ and Its Applications in Neural Networks

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Fudan University & University of Notre Dame

In this paper, we discuss problems of approximation to functions in $L^2(R^n)$ and operators from $L^2(R^{n_1})$ to $L^2(R^{n_2})$ by Radial-Basis Functions. The results obtained in this paper solve the problem of capability of RBF neural networks, a basic problem in Neural networks.

Suppose $g : R^+ \to R^1$. Many papers concerning approximation by functions

$$\sum_{i=1}^{N} g\left(\lambda_i \|X - X_i\|_{R^n}\right)$$

have been published. It is natural to raise the following question: What are the necessary and sufficient conditions imposed on the function g such that the family of functions $\sum_{i=1}^{N} c_i g(\lambda_i \|X - X_i\|_{R^n})$ is dense in $L^2(R^n)$?

The purpose of this paper is to answer this question and construct neural network model, which can be used to approximate nonlinear continuous operators from $L^2(R^n)$ to $L^2(R^n)$. The main results are as follows.

THEOREM 1. *Suppose* $g : R^+ \to R^1, (1 + (|x|)^{\frac{n-1}{2}} g \in L^2(R^+)$, *then the set of linear combinations* $\sum_{i=1}^{N} c_i g(\lambda_i \|X - X_i\|_{R^n})$ *is dense in* $L^2(R^n)$, *where* $\lambda_i > 0, c_i \in R^1, X_i \in R^n, i = 1, \ldots, N$.

THEOREM 2. *Suppose* V *is a compact set in* $L^2(R^n), g : R^+ \to R^1, g(\|X\|_{R^n}) \in L^2(R^n)$, *then for any* $\epsilon > 0$, *there are positive integer* N, *real numbers* $\lambda_i > 0, x_i \in R^n, i = 1, \ldots, N$, *which are independent of* $f \in V$, *and constants* $c_i(f), i = 1, \ldots, N$, *depending on* $f \in V$, *such that*

$$\left\| f(X) - \sum_{i=1}^{N} c_i(f) g(\lambda_i \|X - X_i\|_{R_n}) \right\|_{L^2(R^n)} \leq \epsilon \tag{1}$$

holds for all $f \in V$. *Moreover, all* $c_i(f)$ *are linear continuous functionals defined on* V.

THEOREM 3. *Suppose that* $F \in L^2(R^n)$, *then the family of all linear combinations* $\sum_{i=1}^{N} c_i F(\lambda_i p_i(X) + Z_i)$ *is dense in* $L^2(R^n)$, *where* $c_i \in R^1, \lambda_i \in R^1, Z_i \in R^n, p_i$ *are rotations in* $R^n, k = 1, \ldots, N$.

REMARK 1. *The assumption made in Theorem 1 is equivalent to* $g(\|X\|_{R^n}) \in L^2(R^n)$, *which is also necessary for the validity of Theorem 1.*

Approximation Using Positive Definite Functions

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Positive definite functions (not necessarily radial) can be used to advantage in approximating real-valued functions on $\mathbb{R}^n$ or on S^m. In both cases, we usually require *strictly* positive definite functions, and thus a characterization of these functions is needed at the start. Not only can these functions be used for scattered data interpolation, but they form "fundamental" families in $C(\mathbb{R}^n)$, in $C(S^m)$, and in $C(S^\infty)$. Recent work in this area will be summarized in the talk. In particular, contributions by Xingping Sun, Valdir Menegatto, and Kuei-Fang Chang will be discussed.

Continuous Wavelet Transform on Tangent Bundles of Spheres

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In this paper, the continuous wavelet transforms on $L^2(\mathbf{R})$ and $L^2(\mathbf{R}^2)$ are generalized to TS^1 and TS^2. We consider specific groups that admit square-integrable representations in $L^2(TS^1)$ and $L^2(TS^2)$, respectively. Under the action of these groups, points in the fibers are scaled and translated whereas points in the underlying manifolds are only translated. Modified versions of these groups can be used to define the generalized wavelet transform. It turns out that this transform is a multiple of an isometry.

Multiscale Analysis, Approximation, and Interpolation Spaces

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Striking advancements of mathematical modelling and numerical simulation during the past ten or fifteen years have been to a great extent due to a significant change in the traditional profile of numerical analysis. Whereas classical numerical linear algebra schemes tended to isolate the final problem formulation (often as a prohibitively large linear system of equations) from its origin, a breakthrough in the complexity of solution processes

could be achieved by exploiting possibly much information about the structure of the underlying infinite dimensional continuous problem. The objective of this talk is to indicate some contributions of approximation theory in this context. This can be exemplified for multiscale basis transformations arising in connection with data compression or multilevel solvers for differential or integral equations. The stability of such transformations in a general Hilbert space context can be reinterpreted via the concepts of biorthogonality and Riesz-bases in terms of certain norm equivalences. Tools from interpolation theory, based on classical ingredients like Bernstein and Jackson inequalities, can then be employed to analyse the validity of such norm equivalences. We conclude with briefly indicating some of their consequences and applications.

Approximation Theory for Erdös Weights

Steven B. Damelin

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In recent years the subject of weighted approximation on the real line has received considerable attention with the development of a general theory of orthogonal polynomials for these weights. In this talk, we survey the topic of weighted approximation for Erdös weights with particular emphasis on recent work by D. S. Lubinsky and the author.

Roughly, an Erdös weight is the form

$$W(x) = \exp(-Q(x))$$

where

$$Q : R \to R$$

is of faster than polynomial growth at infinity. The archetypal example being

$$Q(x) = Q_{k,\alpha}(x) = \exp_k\left(|x|^\alpha\right), \quad \alpha > 1, k \geq 1,$$

where

$$exp_k = \exp\left(\exp(\exp(\ldots))\right)$$

is the k^{th} iterated exponential. The results to be presented include a selection from sharp necessary and sufficient conditions for Lagrange interpolation, sharp necessary and sufficient conditions for orthogonal expansions and Jackson–Bernstein approximation theorems.

Better Dual Functions for Gabor Lattices

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(Abstract not available)

A New Method to Determine Regularity of Refinable Functions

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A function f is refinable if it can be written as a linear combination of its translated dilates $f(2x-n)$. Such functions come up in computer aided design, in subband filtering, and in the construction of wavelet algorithms. Several methods have been developed to determine the regularity of f from the coefficients in the refinement equation. Most of these methods are practical only if the linear combination in the refinement equation is finite. In joint work with Albert Cohen, we developed a new method, borrowed from dynamical systems theory, to determine the regularity of refinable functions; this new method works as well for infinitely many as finitely many non-zero coefficients, and it extends to multidimensional non-separable refinement equations.

Characterization of the Best Uniform Approximations of Periodic Functions by Convex Classes Defined by Strictly CVD Kernels

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We say that a kernel $K(x,y) \in C(T^2)$ is *strictly CVD*, if the number of zeros of

$$f(x) = \int_0^{2\pi} K(x,y)h(y)\,dy$$

in a period is less than or equal to the number of cyclic sign changes of h, for any $h \in L^\infty(0,2\pi)$.

According to S. Karlin (*Total Positivity* 1, p. 259), we have

$$\varepsilon_k \det \left\| \int_{I_j} K(\vartheta_i, y)\,dy \right\|_{i,j=1}^{2k-1} > 0,$$

$\vartheta_1 < \cdots < \vartheta_{2k-1} < \vartheta_1 + 2\pi$, $I_1 < \cdots < I_{2k-1} < I_1 + 2\pi$, $\mu(I_j) > 0$, $j = 1, \ldots, 2k-1$, where I_j is an interval, ε_k is $+1$ or -1, $k = 1, 2, \ldots$.

Set

$$\mathcal{M} = \left\{ f(x) : f(x) = \int_0^{2\pi} K(x,y)h(y)\,dy, |h(y)| \le 1 \quad \text{a. e.}, y \in [0, 2\pi) \right\}.$$

We present the following characterization that is the periodic analog of an result of A. Pinkus (*Journal of Approximation Theory* (1981), **33** (2)).

THEOREM. *Assume that $K(x,y)$ is strictly CVD and $g \in C(T) \setminus \mathcal{M}$. The best approximation*

$$f^*(x) = \int_0^{2\pi} K(x,y)h^*(y)\,dy$$

to g from $\mathcal{M}$ is uniquely characterized as follows.

There exists an $n = 1, 2, \ldots$ and knots $\xi_1 < \cdots < \xi_{2n} < \xi_1 + 2\pi$ and $\vartheta_1 < \cdots < \vartheta_{2n} < \vartheta_1 + 2\pi$, such that

$$\det \|K(\vartheta_i, \xi_j)\|_{i,j=1}^{2n} = 0;$$

$$h^*(y) = (-1)^{j+1} \quad \text{a. e.,} \ \xi_j < y < \xi_{j+1}, j = 1, 2, \ldots, 2n;$$

$$\begin{aligned} g(\vartheta_1) - f^*(\vartheta_1) &= -(g(\vartheta_2) - f^*(\vartheta_2)) = \ldots = g(\vartheta_{2n-1}) - f^*(\vartheta_{2n-1}) \\ &= -(g(\vartheta'_{2n}) - f^*(\vartheta'_{2n})) = -(g(\vartheta''_{2n}) - f^*(\vartheta''_{2n})) \\ &= \varepsilon_n \left(\text{sign det} \left\| \int_{\xi_j}^{\xi_{j+1}} K(\vartheta_i, y)\,dy \right\|_{i,j=1}^{2n} \right) \|g - f^*\|_{C(T)} \end{aligned}$$

for some $\vartheta'_{2n}, \vartheta''_{2n}$ satisfying

$$\vartheta_{2n-1} < \vartheta'_{2n} \le \vartheta_{2n} \le \vartheta''_{2n} < \vartheta_1 + 2\pi.$$

Schatten–von Neumann Classes Versus Besov Spaces of Integral Operator's Kernels

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Let $S_p, 0 < p \le \infty$, be the Schatten–von Neumann ideal of compact operators bounded from $H_0 = L_2(\mathbf{R}^m)$ to $H_1 = L_2(\mathbf{R}^n)$ with quasinorm $\|T|S_p\| = (\sum_{k=1}^{\infty} s_k(T)^p)^{1/p}$, where $s_k(T)$ are the s-numbers of T.

Let $\dot{B}^{\sigma}_{pp}(\mathbf{R}^l), \sigma \in \mathbf{R}$, be the homogeneous Besov space with equal metric indices.

Our main result states that the factor-space of S_p modulo composition with unitaries on H_0 and H_1 is isomorphic to $B = \dot{B}_{pp}^{(m+n)(1/p-1/2)}(\mathbf{R}^{m+n})$. In more detail, for every $T \in S_p$ there exists an integral operator T_1:

$$T_1 f(x) = \int_{\mathbf{R}^m} K(x,y) f(y) dy, \; x \in \mathbf{R}^n \; ,$$

such that

1) $T_1 = U_0 T U_1$, where U_j is a unitary on $H_j, j = 0, 1$, and
2) $\|T|S_p\| = \|K|B\|$.

Conversely, every integral operator T with kernel $K \in B$ is in S_p, and there exist $c_1, c_2 : 0 < c_1 \leq c_2 < \infty$, depending on m, n and p only, such that

$$c_1 \|T|S_p\| \leq \|K|B\| \leq c_2 \|T|S_p\| \; .$$

This is a generalization to all $p : 0 < p \leq \infty$ of the well-known theorem about Hilbert-Schmidt operators ($p = 2$)—in which case $c_1 = c_2 = 1$ and the aforesaid isomorphism is an isometry. Best-approximation properties of s-numbers imply that the present result provides direct and converse theorems relating best approximation by finite-rank operators with regularity of the operator's kernel in terms of Besov quasinorms.

The limiting case $p = \infty$ is of extreme importance in itself, since it provides an explicit expression for the uniform operator norm of a compact operator between H_0 and H_1. In particular, the norm of an integral operator is equivalent to the norm of its kernel in $\dot{B}_{\infty\infty}^{-(m+n)/2}(\mathbf{R}^{m+n})$.

Generalization to the case when $H_j, j = 0, 1$, are arbitrary separable Hilbert spaces is an easy application of the standard canonic-isometry argument. Moreover, the most general case when H_0 and/or H_1 is non-separable is easily reduced to the separable case because the image of any compact operator is separable.

There is an immediate application of the main result in obtaining apriori estimates for the Fredholm resolvent $T(\lambda I - T)^{-1}$ and for the solutions of Fredholm integral equations of the second kind in terms of the kernel's regularity. Namely, the well-known Carleman's inequality for $p = 2$ involving the L_2-norm of the kernel can now be extended for all $p : 0 < p < \infty$.

We note that in the present context the general technique of proving a pair of adjusted Jackson- and Bernstein-type inequalities is insufficient, because the spaces X_0 and X_1 in which approximation is being carried out are different from the spaces Y_0 and Y_1 which appear in the respective K-functional. The difficult part of the proof is to show that X_j is isomorphic to $Y_j, j = 0, 1$, and to this end some facts related to unconditional bases of orthogonal wavelets are crucial.

Some Error Estimates on the Classes of Functions for Interpolation by Smooth Chebyshev Generalised Polynomials

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Let $\{\overset{*}{e}_i(\cdot)\}_0^\infty \in C^{(\infty)}[a,b]$ be a ECT-system defining the space of smooth Chebyshev generalised polynomials (STG-polynomials) $\overset{*}{\pi}[a,b] \overset{def}{=} \operatorname{span}\{\overset{*}{e}_i(\cdot)\}_0^\infty$.

It is well-known that for any $x(\cdot) \in C^{(m)}[a,b] (m \geq 0)$, determined by the table of their values in the fixed collection of knots

$$\Delta_n = \{(t_i)_0^n \in [a,b] : (i' \neq i'') \Rightarrow (t_{i'} \neq t_{i''}\} \ \ (n \geq 0),$$

there exists a unique *interpolation STG-polynomial*

$$\overset{*}{\varphi}_n(\cdot) \in \overset{*}{\pi}_n[a,b] \overset{def}{=} \operatorname{span}\{\overset{*}{e}_i(\cdot)\}_0^n \subset \overset{*}{\pi}[a,b],$$

satisfying the following conditions $\overset{*}{\varphi}_n(t_i) = x(t_i) (i = \overline{0,n})$. To estimate the error $r_n = x(t) - \overset{*}{\varphi}_n(t)$ we introduce the following notions:

$V(\cdot; t_{i_o}, \ldots, t_{i_j})$—the STG-polynomials (of Newton's type) with respect to the corresponding collections of points

$$\left\{(t_{i_k})_{k=0}^{j \leq n} \subset \Delta_n : (k' \neq k'') \Rightarrow (i_{k'} \neq i_{k''})\right\};$$

$D_m[x(\cdot)]$—generalized derivative of degree m for $x(\cdot)$ ("by virtue of the ECT-system");

$$W^{(m)}[M_m; a, b] \overset{def}{=} \left\{x(\cdot) \in C^{(m)}[a,b] : \max_t |D_m[x(t)]| \leq M_m\right\}.$$

Then we can prove the following result.

PROPOSITION. *For any $x(\cdot) \in W^{(m)}[M_m; a, b] (0 < m \leq n+1)$ the structural error estimate holds in the form of the inequality*

$$|r_n(t)| \leq \frac{M_m}{\vartheta_m} f_m(t) \quad (a \leq t \leq b) \tag{1}$$

where $\vartheta_m \overset{def}{=} \min_t |D_m[\overset{*}{e}_m(t)]| > 0$, and $f_m(\cdot)$ is independent of $x(\cdot)$ nonnegative function with $f_m(t_i) = 0$ $(i = \overline{O,n})$.

In particular, one can assume that

$$f_{n+1}(t) = |V(t; t_0, \ldots, t_n)|$$

and

$$f_n(t) = 2\left|\prod_{i=0}^{n} V(t; t_0, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)\right|^{1/n+1}. \tag{2}$$

Remark. It follows from (2) that for the classical case

$$f_{n+1}(t) = |(t-t_0)\ldots(t-t_n)|$$

and

$$f_n(t) = 2\,|(t-t_0)\ldots(t-t_n)|^{n/n+1}.$$

Since in this case $\vartheta_m = m!$, then (1) implies (when $m = n+1$ and $m = n$) the well-known error estimates on the classes of functions for interpolation by classical polynomials.

Nonlinear Wavelet Approximation

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We shall discuss some recent aspects of nonlinear wavelet approximation including best basis selection and adaptive pursuit. There is little known in terms of error estimates for these highly nonlinear methods of approximation. We shall discuss what is known and future directions.

Best Approximation and K-Functionals

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The best approximation from a subspace of eigenfunctions of a given differential operator is related to the K-functionals given by that operator. An approximation process is introduced and its rate of convergence is shown to be equivalent to a given K-functional via a strong converse inequality. Application to simultaneous approximation is given.

Best Approximation on Some Function Spaces

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No abstract received.

Orthogonal Polynomials and the Construction of Scaling Functions with arbitrary approximation order

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We use the theory of orthogonal polynomials to construct continuous, compactly supported, orthogonal, spline scaling functions with arbitrary order of approximation.

Non-diagonal Quadratic Hermite–Padé Approximation to the Exponential Function

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Approximation to the exponential function of the form

$$E_{mns}(z) := P_n(z)e^{-2z} + Q_m(z)e^{-z} + R_s(z) = O\left(z^{m+n+s+2}\right),$$

where P_n, Q_m and R_s are polynomials of degree at most n, m and s respectively and P_n has leading coefficient 1 is considered. Explicit formulas and exact asymptotics are obtained for certain ray sequences of $\{E_{mns}\}, \{P_n\}, \{Q_m\}$ and $\{R_s\}$. It is also shown that these Hermite-Padé Type I polynomials can be used to asymptotically minimize expressions of the above form on the unit disk.

On Interlacing Properties of the Roots of Orthogonal and Euler-Frobenius Polynomials

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Ultraspherical polynomials and Euler-Frobenius polynomials are examples of sequences of polynomials generated recursively by the scheme: $P_0(x) = x^\ell$ (ℓ a nonnegative integer) and $c_{n+1}P_{n+1} = T_{r_n}(P_n)$ where $T_r(P)(x) = -2rxP(x) + (1-x^2)DP(x)$. We investigate interlacing properties of the roots of polynomials P_n and P_{n+1} or P_n and P_{n+2}. We obtain similar results for Hermite-like polynomials obtained by the scheme $H_0(x) = x^\ell$ and $c_{n+1}H_{n+1} = T(H_n)$ where $T(H)(x) = -2xH(x) + DH(x)$. Finally, an increasing property of the main root of Euler-Frobenius polynomials is obtained for the most important cases.

Nonstationary Subdivision Schemes and Multiresolution Analysis

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Nonstationary subdivision schemes consist of recursive refinements of an initial sparse sequence with the use of masks that may vary from one scale to the next finer one. The case of masks with supports that grow linearly from one scale to the next is analysed. General sufficient conditions for the convergence of such schemes to C^∞ compactly supported basis functions are derived. Under additional mild conditions, these limit functions allow to define a multiresolution analysis that has the property of spectral approximation. Finally, these general results are used to construct C^∞ compactly supported cardinal interpolants and also C^∞ compactly supported orthonormal wavelet bases that constitute Riesz bases for Sobolev spaces of any order.

The talk presents joint work with Albert Cohen.

Asymptotic Approximation by Cardinal Series

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Using the theory of distributional asymptotic expansions, we obtain asymptotic approximations of functions by series of cardinal type,

$$\sum_{n=-\infty}^{n=\infty} f(n\varepsilon)g(x/\varepsilon - n), \quad \text{as } \varepsilon \to 0,$$

as well as by series of quasi-interpolants, for a wide class of functions f and g.

Hermite Interpolation with Radial Basis Functions on the Sphere

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We show how conditionally negative definite functions on the sphere can be used for Hermite interpolation. These results are obtained by combining ideas of [Menegatto '94] and [Narcowich '94], as well as classical concepts of Schoenberg. We also conclude that the basis functions we use are natural in the sense that Hermite interpolation can be viewed as the limit of an associated Lagrange interpolation problem, thus justifying some observations made by [Franke, Hagen, and Nielson '93].

On An Algebraic Modulus of Smoothness and Best Algebraic Approximation

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We consider the space of continuous functions on $[-1,1]$ as well as the L^p space with weight $\frac{1}{\sqrt{1-x^2}}$ and introduce a modulus of smoothness that is based on the algebraic addition $a \oplus b := a\sqrt{1-b^2} + \sqrt{1-a^2}b$ defined on $[-1,1]$.

It will be shown that the difference operator of higher order behaves completely different near the endpoints ± 1 than the corresponding classical difference operator for periodic functions. This is due to the fact that $([-1,1], \oplus)$ is not a group because the associative law is missing around the boundaries.

Steklov means and an equivalent K-functional are given. Moreover the equivalence with the Butzer-Stens modulus, introduced by the Chebychev translation will be seen.

Classification of All Function Systems in L_p and the Place of Wavelets and Prewavelets in this Classification

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The interest to the wavelets is the natural development of some part of function theory, which has connection with concrete function systems (trigonometric, Haar, etc.). It concerns applied using of these systems. Since there exists the infinite number of function systems, the question on optimization of some function systems (maybe classical) appears for some problems. To answer on these questions we must somehow classify all function systems (maybe the same as living world of Earth).

That will make time quick for the solution of the questions of optimizing. The properties of function systems depend on function spaces (for example there is no basis in $L^p(0,1), 0 < p < 1$). Therefore, it is natural to consider the function systems in the spaces: $a) L^p(0,1), 0 < p < 1; b) L^p(0,1), 1 \le p < \infty$, (if we limit ourselves by the scale of $L^p, p > 0$). But it is more natural to consider the systems in all subspaces of the space S, almost everywhere finite measurable functions (for example the scale of the Orlicz spaces) The results given below are more general than for wavelets in the paper [1] . It is the first step to the classification of all function systems.

DEFINITION. *A system of $\{f_n\}_{n=1}^{\infty} \subset L^p, 0 < p < \infty$ is called a representation system in the space L^p if for any $f \in L^p$ there exists a series $\sum_{k=1}^{\infty} c_k f_k$ such that*

$$\lim_{n\to\infty} \|f - \sum_{k=1}^{n} c_k f_k\|_{L_p} = 0.$$

Let $supp\varphi_n = \{x : \varphi_n(x) \neq 0\}$; $x_n = inf\{x \in [0,1] : \forall \epsilon > 0, mes\{(x, x+\epsilon) \bigcap supp\varphi_n\} \neq 0\}$; $y_n = sup\{y \in [0,1] : \forall \epsilon > 0, mes\{(y-\epsilon, y) \bigcap supp\varphi_n\} \neq 0\}$;

Denote $d(\varphi_n) = y_n - x_n$; $Q_n = [x_n, y_n]$. We consider the systems $\{\varphi_n\} \subset L^p(0,1), p \ge 1$, satisfies the conditions:

(1) $d(\varphi_n) \to 0$;
(2) $\int_0^1 \varphi_n(x)dx \neq 0$;
(3) $\forall N \in \mathbf{N}\ mes\{\bigcup_{k=N}^{\infty} Q_k\} = 1$ let $\sigma_n = inf_{\lambda \in R} \|(1 - \lambda\varphi_n(x))\chi_{Q_n}(x)\|_p (\frac{1}{d(\varphi_n)})^{\frac{1}{p}}$. From Lemma 1 [1] it follows that, if the (2) are satisfied, then $\sigma_n < 1$.

THEOREM 1. *Let for a subsystem $\{\varphi_{n_k}(x)\}$ of the system $\{\varphi_n\}$ the conditions (1), (2) and $sup_{n_k} \sigma_{n_k} < 1$ are satisfied. Then this subsystem $\{\varphi_{n_k}\}_{k=N}^{\infty}$, where $N \in \mathbf{N}$ is any number, is a representation system in $L^q(0,1), q \le p$, if and only if the condition (3) is fulfilled.*

There are examples for non-improving of assumptions (1)–(3) in Theorem 1. If the assumption (3) is not satisfied, then the system $\{\varphi_n\}$ is not a full system. If the assumption (2) or (1) is not satisfied then there exist the examples, when the system $\{\varphi_n\}$ is not a full one.

Remark If the system $\{\varphi_n\}$ consists of subsystems, which satisfy the assumptions in the II level or III level and etc., then we must consider all these subsystems.

[1] V. Filippov, P. Oswald. Representation in L^p by series of translates and dilates of one function, J. Appr. Theor. (to appear).

On Best Linear Approximation in $l_1(n)$

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Consider the space $l_1(n)$; i.e., $\mathbf{R}^n$ endowed with $|\cdot|_1$, and let ${}_1P_U : \mathbf{R}^n \to 2^U$ be the metric projection of $l_1(n)$ onto an r-dimensional linear subspace U. The subspace and its *metric complement* $U^{(0)} := {}_1P_U^{-1}(0)$ decompose (in general not uniquely) $\mathbf{R}^n$; i.e.,

$$\mathbf{R}^n = U^{(0)} + U.$$

The author explicitly describes the metric complement: Let $R := U^\perp \cap \overline{b_1^\infty(0)}$ denote the intersection of the orthogonal complement of U with the closed dual unit ball of $l_\infty(n)$; R is a polyhedron of dimension $n-r$. The faces of the polyhedron are considered to be relatively open. Each face of R is uniquely determined by a point of its relative interior, e.g., by its center of gravity w, and denoted by $face_w$; obviously, $\operatorname{rel\,bd} R = \dot{\bigcup}_{face_w \subset R} face_w$.

Let $S_U := U^{(0)} \cap S_1(0)$ denote the intersection of $U^{(0)}$ with the $l_1(n)$-unit sphere the so-called *Schattengrenze* of U; in other words, $U^{(0)} = \operatorname{cone}(S_U; 0)$. Each face $face_w$ of R determines a face S_w of $S_1(0)$ belonging to the Schattengrenze; in particular,

$$S_U = \bigcup_{w \in \operatorname{ext} R} \overline{S_w}.$$

Introducing $K_w = \operatorname{cone}(S_w; 0) + U$, which is for $w \in \operatorname{ext} R$ an open cone with vertex in 0, leads to a decomposition of $\mathbf{R}^n$ into pairwise disjoint cones; i.e.,

$$\mathbf{R}^n \setminus U = \dot{\bigcup_{face_w \subset R}} K_w, \qquad \text{and} \qquad \mathbf{R}^n = \bigcup_{w \in \operatorname{ext} R} \overline{K_w}.$$

In particular, if the metric projection onto U is unique, the decomposition is unique:

$$\mathbf{R}^n = U^{(0)} \oplus U \quad \text{and} \quad K_w = \operatorname{cone}(S_w; 0) \oplus U \quad \text{for all } face_w \subset R.$$

In this case the metric projection is linear on each of the cones $\overline{K_w}$, $w \in \operatorname{ext} R$. For an arbitrary U these considerations lead to a selection of the metric projection which is characterised by the following properties:

- the error $\delta = x - u^*$ has minimal support for the selected $u^* \in {}_1P_U(x)$;

- the selection is piecewise linear, and there is a finite number of cones of linearity decomposing $\mathbf{R}^n$; in particular, the selection is globally Lipschitz continuous;
- w.r.t. these properties the selection is in a way "closest" to the strict approximation in $l_\infty(n)$.

Orthogonal Polynomials and Solving Linear Systems of Equations

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In this note we discuss the connection between orthogonal polynomials and the solution of linear systems of equations by polynomial iteration methods.

To this end, let $\langle \cdot, \cdot \rangle$ denote an (appropriate) inner product for the space Π_n of polynomials of degree n. The design of an effective polynomial iteration method leads to approximation problems of the form

$$\langle p_n, p_n \rangle = \min\{\langle p, p \rangle : \ p \in \Pi_n, \ p(0) = 1\},$$

or to problems like

$$\langle p_n, p_n \rangle = \min\{\langle p, p \rangle : \ p \in \Pi_n, \ p(0) = 1, \ p'(0) = 0\},$$

respectively. We note that the latter problem is in particular interesting in connection with singular systems.

The solution of the first problem is given by the Kernel polynomials with respect to $\langle \cdot, \cdot \rangle$. Also, for the second problem the unique solution is explicitly known and may be called Hermite Kernel polynomial. We will characterize the Hermite Kernel polynomial and we will devise (stable) recurrence relations for the computation of these polynomials. Furthermore, we will point out relationships between the two mentioned polynomial classes and we will outline the connection to the actual implementation of polynomial iteration methods.

Plane Preserving Volume Deformations Using Scattered Landmark Points

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We address the problem of deforming one three dimensional volume to another volume. For our applications, landmark points in the domain volume are arbitrarily located and it is desired to construct a deformation to another volume so that the scattered landmark

points get mapped to another set of corresponding points in the range volume. Applications involve modeling solids, volumetric morphing and mapping one brain volume to a "standard" brain volume. For the brain deformation, we modify the interpolant so that the separating plane in the first brain is also mapped to the separating plane of the second brain.

Computation of Padé Approximations of Transfer Functions via the Lanczos Process

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The behavior of time-invariant single-input single-output linear dynamical systems is characterized by their frequency-domain transfer functions. Although these functions are rational, their numerator and denominator polynomials are of very high degree if the underlying system is large, and then it becomes prohibitive to compute the transfer function. Instead, for large systems, one approximates the transfer function by a rational function of lower order. A common choice is to employ Padé approximations, which are computed in the usual fashion by first generating the moments, followed by a solution of a Hankel system. Unfortunately, this approach is numerically unstable due to the typical extreme ill-conditioning of the Hankel system, and it can only be used to compute Padé approximants of very moderate order.

In this talk, we present an alternative algorithm for the stable and efficient computation of Padé approximations of transfer functions. Exploiting the connection between Padé approximation and the Lanczos process for the successive tridiagonalization of large matrices, our algorithm generates the Padé approximants directly from the tridiagonal Lanczos matrices, and it thus avoids the ill-conditioned moment computations. We present numerical results for a variety of linear systems arising in the simulation of large electronic circuits.

For the general case of multi-input multi-output linear dynamical systems, the associated transfer functions are matrix-valued rational functions. We also describe an algorithm for the computation of matrix Padé approximants to such matrix-valued transfer functions. The algorithm is based on a new Lanczos process for multiple starting vectors.

Part of this work is joint with Peter Feldmann, AT&T Bell Laboratories.

Relations Between Best Approximations

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Let $E(f, B, F)$ be the best approximation of an element f by elements from B in the metric of a normed space F. Relations between best approximations $E(f, B, F)$ by different subspaces B play an important role in Approximation Theory. Recently the author has developed a general approach for obtaining such relations, including the cases of approximations by algebraic and trigonometric polynomials and entire functions of exponential type (Bernstein, Ganzburg), of approximations by trigonometric polynomials and spline functions (Velikin), of rational and spline approximations (Petrushev), and of weighted polynomial approximations with different weights.

These results have different applications in Approximation Theory such as multidimensional Jackson theorems, multidimensional Bernstein-, Nikolskii-, and Remez-type inequalities, sharp constants of multidimensional approximations and others.

In this talk we shall present a survey of these problems and give new relations between best approximations. In particular, some results are extended to the case of quasinormed spaces F. For instance, for $0 < p < 1$,

$$\lim_{n\to\infty} E(f, \mathcal{P}_{n,m}, L_p(nV^*)) = E(f, B_V, L_p(R^m)), \tag{1}$$

where R^m is the m-dimensional Euclidean space, V is a centrally symmetric convex compactum in R^m, V^* is the polar of V, B_V is the class of entire functions of exponential type whose spectrum is contained in V, $\mathcal{P}_{n,m}$ is the class of polynomials in m variables of degree n, $L_p(\Omega)$ is the space with the quasinorm $||f||_{L_p(\Omega)} = (\int_\Omega |f|^p dx)^{1/p}$, $0 < p < 1$.

For $1 \le p \le \infty$ the relations like (1) have been obtained by Bernstein, Raicin and Ganzburg. This relation is based on Remez- and Nikolskii-type inequalities for multidimensional polynomials in symmetric quasinormed spaces.

Approximation by Separable Functions

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The approximation by separable functions is one of the topics in Approximation Theory and Numerical Analysis where Ward Cheney has contributed many important results during the last 20 years.

Separable Functions are defined as follows. Let $S \subset \mathbf{R}$, $T \subset \mathbf{R}$ and $D \subset S \times T$ be compact sets. Two finite-dimensional subspaces $U \subset \ell_\infty(S)$ and $V \subset \ell_\infty(T)$ of bounded functions, with basis $(u_j)_1^m$ or $(v_k)_1^n$, respectively, generate the linear space $W_\infty(D)$ of separable functions

$$(1) \qquad w(s,t) = \sum_{j=1}^{m} u_j(s)x_j(t) + \sum_{k=1}^{n} y_k(s)v_k(t), \quad (s,t) \in D,$$

where $x_j \in \ell_\infty(T)$ and $y_k \in \ell_\infty(S)$ are arbitrary bounded functions. In the applications, S, T and D are finite sets and U and V are polynomial or spline spaces.

Two problems will be discussed:

(i) Given $f \in \ell_\infty(D)$ or $f \in C(D)$, does there exist a best approximation $w^D(\cdot, f)$ of f from $W_\infty(D)$ in the uniform norm $\|\cdot\|_D$ on D ?

(ii) If Δ is a (finite) subset of D and $w^\Delta(\cdot, f)$ is a best approximation of f from $W_\infty(\Delta)$ in the uniform norm $\|\cdot\|_\Delta$ on Δ, then compare the distances

$$\|f - w^\Delta\|_\Delta, \quad \|f - w^\Delta\|_D, \quad \|f - w^D\|_D.$$

Approximations by Periodic Biorthogonal Wavelets: Numerical Algorithms

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In this paper we develop fast numerical algorithms for representations of 1-periodic functions in biorthogonal multiresolution analyses. The algorithms include decomposition, reconstruction, interpolation, point evaluation and representation of differential operators having constant coefficients. Since all these operations are of convolution type, it is natural to make use of fast Fourier transform technique. We show that in the Fourier domain the decomposition and reconstruction algorithms have a matrix representation in terms of permutations and of 2×2 block diagonal matrices. For representation of derivatives, linear systems with circulant matrices must be solved which can be done directly using discrete Fourier transform. One important fact about biorthogonal multiresolution analyses is that they are compatible with derivatives. We show that this property can be used for easy computations of stiffness matrix elements. It also helps in the study of the Gibbs phenomenon in approximations of functions having jump discontinuities in some of their derivatives. We present some illustrative examples using biorthogonal spline wavelets.

Parallel Wavelet-Galerkin Methods by Means of Adapted Wavelet Packet Bases

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Wavelet approximation methods for the numerical solution of differential and pseudodifferential equations have been recently investigated as a powerful tool for preconditioning and compressing the stiffness matrices relative to orthogonal and biorthogonal wavelet bases [1], [2], [3]; moreover stability and convergence results have been established for a very general setting [4].

In recent studies, new compactly supported biorthogonal wavelet bases have been constructed, which are adapted to some simple differential operators [5]; on these bases the stiffness matrices have a simple structure and bounded condition number. Looking for an efficient parallel algorithm for the numerical solution of the Galerkin approach, in this paper we propose the use of compactly supported wavelet packet bases adapted to quite general, constant coefficient, elliptic differential operators. We show how these bases can be evaluated by means of simple convolution operations of finite support vectors and we study, specially from a numerical point of view, the consequences of using these bases for the parallel solution of simple, constant coefficient model problems. Performance and efficiency of the parallel algorithm are estimated from a first experimentation on a distributed memory hypercube multiprocessor.

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[4] W. Dahmen, S. Proessdorf and R. Schneider, Wavelet approximation methods for pseudodifferential equations I: Stability and convergence, IAAS-Preprint No.7, Berlin 1992.

[5] S. Dahlke, I. Weinreich, Wavelet-Galerkin methods: An adapted biorthogonal wavelet basis, *Const. Approx.* **9** (1993), 237–262.

Error Bounds for Linear Interpolation on a Triangle

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We attempt to determine best possible bounds for the errors in function value and derivative when a function is approximated by linear interpolation between values at the

vertices of a triangle of known shape and size. We consider both L_2 and L_∞ bounds, in terms of L_2 and L_∞ measures of smoothness of the function.

Orthogonal Multiwavelet Constructions in 2D

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In the usual construction of a wavelet using a multiresolution analysis (MRA) (V_p) is a single function φ called the scaling function whose set of integer translates forms a Riesz basis for V_0. In contrast multiwavelets are constructed from an MRA that is generated by a finite set of scaling functions $\varphi^1, \ldots, \varphi^r$. If there is some set of compactly supported scaling functions whose integer translates form an orthogonal basis for V_0 then (V_p) is called an *orthogonal* MRA.

The authors and co-worker Massopust constructed the first nontrivial continuous, compactly supported, orthogonal multiwavelets using univariate fractal interpolation functions. These multiwavelets have several properties not possible in the single scaling function case. For instance, they are symmetric, interpolatory and remain orthogonal when restricted to integer intervals.

In recent work by the authors, a general scheme for constructing multivariate wavelets was developed. This scheme is based on enlarging a given multiresolution analysis (usually a classical spline space) by adding certain functions to obtain an orthogonal MRA.

This work is extended to the multivariate setting, in particular we add continuous "fractal interpolation surfaces" that are supported on triangles to the MRA generated by the C_0 piecewise linear hat function on a given lattice to get an orthogonal MRA. As in the univariate case the resulting scaling functions are symmetric (with respect to rotations leaving the lattice invariant), interpolatory and remain orthogonal when restricted to triangles in the associated triangulation.

Peano Kernel for Harmonicity Differences of Order p

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We consider linear functionals L which vanish on polyharmonic functions of a fixed order $p \geq 1$ in a bounded domain $D \subset \mathbf{R}^d$, *i.e.*, on the kernel of the polyharmonic operator Δ^p, where Δ denotes the Laplacian differential operator. For such functionals we prove a *theorem of Peano type* which states that

$$L(f) = \int_D \mathcal{P}(x)\Delta^p f(x)dx \qquad (f \in C^{2p}),$$

where $\mathcal{P}$ is the corresponding Peano kernel.

This formula shows a full analogy with the classical one-dimensional case in which

$$L(g) = \int_a^b \mathcal{P}_0(t) g^{(p)}(t) dt \qquad (g \in C^p),$$

and where L is a functional vanishing on the polynomials of degree $p-1$, *i.e.*, on the kernel of the operator $\frac{d^p}{dt^p}$.

It is a well-known result that the Peano kernel $\mathcal{P}_0$ of the one-dimensional finite difference of order p is a univariate B-spline.

We introduce the concept of the *harmonicity difference of order p* and study properties of the Peano kernel associated with it. The harmonicity difference of order p naturally arises from the Pizzetti mean value formula for polyharmonic functions of order p.

The *harmonicity difference of order p* enjoys many of the properties of the one-dimensional finite difference. In particular, we prove that its Peano kernel is (i) compactly supported, (ii) radially symmetric with high order smoothness, and (iii) satisfying an extremal problem which resembles Holladay's theorem for one-dimensional splines.

Our paper is joint work with O. Kounchev (Sofia).

The Orthogonality of Weighted Faber Polynomials

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Weighted Faber polynomials $\{F_n(z; g; E)\}_{n=0}^{\infty}$ associated with a domain E and a weight function g play a very important part in the study of the asymptotic properties of orthogonal polynomials in the complex domain. In the present paper, we study the orthogonality of these polynomials, mainly in dependence on the weight function g and the structure of the domain E. We introduce a class of m-fold orthogonal polynomials and investigate their properties. Our results include certain classical orthogonal polynomials and ordinary Faber polynomials as special cases.

Spline Interpolation and Its Wavelet Analysis

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In this paper, we will discuss the spline interpolants at arbitrary knots with the highest possible approximation order and the corresponding wavelet functions. Considering the spline interpolants as window functions, we will also discuss the corresponding compact

window Fourier transform. In addition, an approach to the corresponding orthogonal scaling function will be also considered.

Existence and Accuracy for Matrix Refinement Equations

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Refinement equations are functional equations of the form $f(x) = \sum_{k=0}^{N} c_k f(2x-k)$. They play a key role in wavelet theory and in subdivision schemes in approximation theory. The coefficients c_k are usually real or complex numbers. Recent "multiwavelet" constructions require matrix c_k and vector-valued $f = (f_1, \ldots, f_r)$. We consider the existence, uniqueness, and order of approximation of compactly supported solutions to matrix refinement equations. Several classes of solutions are identified, depending on whether the infinite matrix product $(\prod_{j=1}^{\infty} M(2^{-j}\gamma))$ converges, possibly in a weak or constrained sense, where $M(\gamma) = \frac{1}{2}\sum c_k e^{-2\pi i k\gamma}$. This behavior is determined by the eigenvalues of $M(0)$. We also show how to calculate the order of approximation p, i.e., the highest degree polynomial $1, x, \ldots, x^{p-1}$ that can be written exactly as a linear combination of the integer translates of $f_1, \ldots, f_r$.

Stability and Linear Independence of the Shifts of A Multivariate Refinable Function

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Due to their so-called time-frequency localization properties, wavelets have become a powerful tool in signal analysis and image processing. Typical constructions of wavelets depend on the orthogonality or stability of the shifts of an underlying refinable function. In this paper, these properties are characterized in terms of the refinement mask for a large class of multivariate compactly supported refinable distributions. Linear independence (which is related to stability) of the shifts is also treated. Among the functions to which these results apply are tensor products and box splines.

Optimal Triangulations Using Edge Swappings

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This is joint work with C. K. Chui. The main result of this paper is an efficient method for triangulating any finitely many scattered sample sites such that these sample sites are the only vertices of the triangulation and that for any discrete data given at these sample sites, there is a C^1 piecewise quartic polynomial on this triangulation that interpolates the given data, with the optimal order of approximation.

Recovering Band Limited, Weakly Stationary Process from a Set of Its Irregularly Spaced Samples

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The problem of recovering, say, a band limited weakly stationary process from a set of its irregularly spaced samples is studied. For rather general sampling sequences some sufficient conditions ensuring mean square or pathwise reconstruction are obtained. For the cases of regular samples with either finitely many missing ones and/or finitely many irregular ones a necessary and sufficient condition is presented. Some elements of the proofs involve classical results on non-harmonic Fourier series as well as more recent results on frames.

Convex Spline Approximation in L_p Space

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Let $f \in L_p[-1, 1]$, $0 < p < \infty$, be convex. Then for any given partition $T_n := \{-1 := x_0 < x_1 < \ldots < x_n := 1\}$ of the interval $[-1, 1]$, there is a convex continuous piecewise quadratic function s_n on T_n with approximation error ω_3, either measured on $[-1, 1]$ or on local subintervals. This function can be smoothed into a C^1 or C^2 (cubic) spline with the same degree of approximation.

REMARK: This is our first step of constructing convex polynomial approximation in L_p with degree of approximation ω_3^φ.

Generalizations of Dykstra's Algorithm

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This talk covers two generalizations of Dykstra's Algorithm. Dykstra's algorithm is a method for computing the best approximations to a set C which is the intersection of a finite number of closed convex sets C_i. The best approximation is found by an alternating projections scheme. This scheme is useful when the projection onto the *individual* sets is easily computed, but the projection onto C is unknown or difficult to compute. The first generalization of this algorithm presented allows the programmer to change the order in which the projections are performed. These changes can be used to accelerate convergence. The second generalization allows C to be an intersection of an infinite sequence of closed convex sets. Since all convex sets in a separable Hilbert space can be represented as the intersection of a countable number of half spaces, this generalization could be useful. Examples of both generalizations and some simple applications will be given.

Scattered Data Reconstruction and Radial Basis Function Interpolation Methods

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We assert to reconstruct a finite number of scattered data $T_1 f, \ldots, T_N f$ that are generated by actions of linear functionals $T_1, \ldots, T_N$ on a function $f : \mathbf{R}^d \to \mathbf{R}$ from a function space to be specified. A possible specialization of our method could be multivariate interpolation. For this case, radial basis functions provide a prominent tool.

Here, we generalize radial basis function interpolation methods and assert to find a *reconstruction function* s_f with respect to the reconstruction conditions $T_j f = T_j s_f$, $(1 \leq j \leq N)$. Due to the reconstruction scheme, s_f is assumed to be a linear combination of translates of a prescribed conditionally positive definite function ψ exceeded by a polynomial from $\mathcal{P}_m^d$, which denotes the linear space of all d variate polynomials of order m or less.

Certainly, the reconstruction functions as described above form a linear space $\mathcal{F}_\psi$ depending on ψ. We assert to topologize $\mathcal{F}_\psi$ by a seminorm $|\cdot|_\psi$ which is intimately related to the conditional positive definiteness of ψ. Then, $(\mathcal{F}_\psi, |\cdot|_\psi)$ is referred as *native function space* of the reconstruction method.

This talk provides a characterization of the native function spaces and relates different terms of conditional positive definiteness.

Wavelet Smoothing Techniques and Interactive Surface Modeling

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Given a set of measured data, we discuss several different wavelet based techniques to quickly fit a smooth surface to the points. Further, given a surface, we present a method, based on a version of Nash-Moser techniques combined with wavelets, to edit the surface to obtain a new one with desired, user-defined properties.

A Generalization of de Boor's Stability Result and Symmetric Preconditioning

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We provide a symmetric preconditioning method based on weighted divided differences which can be applied in order to solve ill-conditioned scattered-data interpolation problems in a stable way. Concerning the theoretical background, an a priori unbounded operator on $\ell_2(Z)$ is preconditioned so as to get a bounded and coercive operator. The method has another interpretation in terms of constructing certain Riesz bases of appropriate closed subspaces of $L_2(R^d)$. In extending Mallat's multiresolution analysis to the scattered data case, we construct nested sequences of spaces giving rise to orthogonal decompositions of functions in $L_2(R^d)$; in this way the idea of wavelet decompositions is (theoretically) carried over to scattered-data methods.

This is joint work with J. Stöckler, University of Duisburg.

Refinable Shift-invariant Spaces: From Splines to Wavelets

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In this talk we shall survey some recent results concerning refinable shift-invariant spaces. A linear space S of (measurable) functions on $\mathbf{R}^s$ is said to be shift-invariant if it is invariant under integer translation, that is,

$$f \in S \Longrightarrow f(\cdot - j) \in S \qquad \forall j \in \mathbf{Z}^s.$$

The space S is said to be refinable if

$$f \in S \Longrightarrow f(\cdot/2) \in S.$$

Refinable shift-invariant spaces form a natural framework for the study of splines and wavelets.

In this talk we cover five topics. First, we review some basic properties of shift-invariant spaces such as stability, linear independence, and orthogonality, and discuss characterizations of shift-invariant spaces. Second, we give a unified treatment to various approximation schemes such as orthogonal projection, bi-orthogonal projection, cardinal interpolation, and quasi-interpolation. We also provide an insight into some recent results on characterization of the approximation order of a shift-invariant space. Third, we study refinement equations and subdivision schemes. We are especially concerned with the existence and smoothness of the solution of a refinement equation, and the rate of convergence of the corresponding subdivision scheme. Fourth, we investigate multiresolution and construction of wavelets. The emphasis is placed on construction of multivariate wavelets which are not tensor products of univariate wavelets. Finally, we examine results on approximation by wavelets. In particular, the direct theorem (Jackson estimates) and the inverse theorem (Bernstein estimates) are discussed in the setting of Besov spaces.

Some Inequalities for Derivatives

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Some inequalities for derivatives will be discussed.

Wavelet Analysis of Speech Signals

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This paper describes experiments in which speech signals were analyzed using several types of wavelets: Haar, Daubechies, spline, Bagor, and chirp. We constructed wavelet sonograms of time versus resolution space and compared them with those from a Fourier method. Since wavelet analysis showed promise for identifying some speech events, follow-up experiments were conducted to assess the viability of wavelet-based tools in speech recognition systems. We will present our findings and discuss possible directions for future research.

A Variational Approach to Subdivision

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Interpolatory subdivision is a technique to iteratively produce smooth curves or surfaces. This is done by inserting new points into a given control polygon or control net. As more and more points are inserted, the sequence of polygons or nets converges to a smooth limit.

In the literature, subdivision schemes are usually defined by an *explicit* rule how to compute the new points in every step. In our approach, however, we *implicitly* define the new points by choosing an energy functional which is to be minimized. Although this in general leads to global dependencies between new and old points, the resulting schemes can be computed with linear time complexity due to the banded structure of the equation systems that have to be solved.

We give general conditions for the existence of solutions and present some examples for such schemes which produce limit curves of arbitrary smoothness. There are several ways to modify this approach.

Geometric Hermite Interpolation

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Since parametric representations of curves are not unique, the approximation rates by splines can be significantly improved. This surprising fact was first observed in [DeBoor-Hoellig-Sabin:1987] where a 6-th order accurate cubic interpolation scheme for planar curves has been constructed. We describe a natural generalization of standard Hermite interpolation for space curves. In addition to position and tangent direction at the endpoints a third point within the parameter interval is interpolated. The resulting method is easy to implement and achieves the optimal approximation order 5. This is a substantial improvement over the order 4 of standard Taylor approximations. The high accuracy of the method is confirmed by examples which also demonstrate a good qualitative behavior of the approximations. In particular the curvature plots show that the shape of the interpolated curves is in general very well preserved.

1. C. de Boor, K. Höllig, and M. Sabin, High accuracy geometric hermite interpolation, *Computer Aided Geometric Design*, 4 (692), 269–278, April 1987.

On the Laguerre Polynomials of Negative Index

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The Laguerre polynomials of index $\alpha > -1$, $\{L_n^\alpha(x)\}$, are the unique polynomials orthogonal on $(0,\infty)$ with respect to the weight $x^\alpha e^{-x}\,dx$ which satisfy the requirements that each L_n^α is a polynomial of exact degree n and the coefficient of x^n is $(-1)^n/n!$. They are defined for $\alpha \leq -1$ using their explicit representation for $\alpha > -1$ and analytic continuation. We prove pointwise convergence for expansions of certain functions in terms of the Laguerre polynomials of index $\alpha \leq -1$. The idea is that although the kernel polynomial cannot be integrated against the weight $x^\alpha e^{-x}$, it behaves as an approximation to the identity provided we apply it to functions which are well behaved near zero or if we subtract from the product of the kernel polynomial and the function a suitable polynomial.

Asymptotically Best L_q-approximation by Splines and Asymptotically Best Quadrature Formulae

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By an appropriate choice of knots and data points for the quasi-interpolants of de Boor and Fix, asymptotically best spline approximants to a given smooth function g can be obtained, where the underlying norm is the L_q-norm with $1 \leq q < \infty$. This method can be used to construct Peano kernels for asymptotically best quadrature formulae. Instead of using these Peano kernels, the quadrature formulae resulting in this way can also be obtained simply by replacing the integrand f by some piecewice polynomial interpolant (this works for integrals with arbitrary integrable weight functions).

On k–monotone Polynomial and Spline Approximation in L_p, $0 < p < \infty$ Metric

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Let Π_n denote the set of all algebraic polynomials of degree $\leq n$, $\mathcal{S}(r,N)$ be the space of all splines of order r with knots $\{i/N\}_{i=0}^N$, $\omega_m(f,\delta)_p$ denote the usual integral modulus of smoothness, and let Δ^k be the set of all functions f such that $\sum_{i=0}^k (-1)^{k-i}\binom{k}{i} f(x+ih) \geq 0$ for all x and h such that $[x, x+kh] \subset [0,1]$. In particular, Δ^1 is the set of all nondecreasing functions. Also, $f \in C^k[0,1] \cap \Delta^k \iff f^{(k)}(x) \geq 0$, $x \in [0,1]$.

The following theorem, which is a generalization (in the case $1 \leq p < \infty$) and a complement ($0 < p < 1$) of a well known result of A. S. Shvedov, is discussed.

THEOREM 1. *Let $k \in N$ and $0 < p < \infty$ be fixed, $m \in N \cup \{0\}$, and let $\nu \in N$ be such that $\max\{k + 2 - m, k\} \leq \nu < k + p^{-1}$. Then for any $n \in N$ and $A > 0$ there exists a function $f \in C^{\infty}[0,1]$, $f^{(k)}(x) \geq 0$, $x \in [0,1]$ such that for every $P_n \in \Pi_n$, $P_n^{(k)}(0) \geq 0$ the following inequality holds*

$$\|f - P_n\|_p > A\omega_m(f^{(\nu)}, 1)_p.$$

COROLLARY. (monotone polynomial and spline approximation). *The estimates*

$$E_n^{(1)}(f)_p \leq C\omega_2(f', 1)_p$$

and

$$\mathcal{E}_{r,n}^{(1)}(f)_p \leq C\omega_2(f', 1)_p$$

are not true in general for $0 < p < \infty$ and $f \in C^{\infty}[0,1] \cap \Delta^1$, where $E_n^{(1)}(f)_p := \inf_{P_n \in \Pi_n \cap \Delta^1} \|f - P_n\|_p$ and $\mathcal{E}_{r,n}^{(1)}(f)_p := \inf_{s \in \mathcal{S}(r,n) \cap \Delta^1} \|f - s\|_p$.

It is interesting, that in the case $p = \infty$ these estimates are true. In fact, the following is valid:

if $f \in C^1[0,1] \cap \Delta^1$, then $E_n^{(1)}(f)_{\infty} \leq Cn^{-1}\omega_m(f', n^{-1})_{\infty}, m \geq 1$ (I. A. Shevchuk, and X. M. Yu and Y. P. Ma), and $\mathcal{E}_{r,n}^{(1)}(f)_{\infty} \leq Cn^{-1}\omega_{r-1}(f', n^{-1})_{\infty}, r \geq 2$ (D. Leviatan and H. N. Mhaskar).

Theorem 1 shows that the estimates in the following theorem are exact in the sense of the orders of moduli of smoothness.

THEOREM A. (monotone spline approximation). *Let $L_p^j[0,1]$, $1 \leq p < \infty$ denote the space of functions which are j-fold integrals of $L_p[0,1]$ functions. Then*

$$\mathcal{E}_{r,n}^{(1)}(f)_p \leq Cn^{-1}\omega(f', n^{-1})_p,$$

if $f \in L_p^1[0,1] \cap \Delta^1$ and $r \geq 2$ (C. K. Chui, P. W. Smith and J. D. Ward) and

$$\mathcal{E}_{r,n}^{(1)}(f)_p \leq Cn^{-2}\omega_{r-2}(f'', n^{-1})_p,$$

if $f \in L_p^2[0,1] \cap \Delta^1$ and $r \geq 3$ (D. Leviatan and H. N. Mhaskar).

Generalized Exponential Box Splines

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Up to a constant factor, the s-variate exponential box spline of Ron, Dahmen, and Micchelli is the convolution of several distributions M of the form

$$\langle M, \varphi \rangle = \int_{x_0}^{x_n} M(t \,|\, x_0, \ldots, x_n) \varphi(\kappa t)\, dt$$

where φ is an s-variate test function, κ is a vector in $\mathbf{R}^s$, and $M(t \,|\, x_0, \ldots, x_n)$ is the univariate B-spline with equidistant knots $\{x_0, \ldots, x_n\}$. In this talk, we will discuss the compactly supported Generalized Exponential Box Spline (or GEB-spline) obtained by letting $\{x_0, \ldots, x_n\}$ have variable stepsize.

The recurrence relations for GEB-splines closely resemble those of the exponential box spline and univariate B-spline.

Taking for a basis a set of shifts of finitely many GEB-splines, we obtain a spline space with a triangulation of periodic stepsize in each of its directions.

Results concerning a partition of unity, the exponential-polynomials contained in this space, and the linear independence of these shifts all generalize corresponding results for the exponential box spline.

Bivariate Splines on FVS-triangulation

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We survey the study of bivariate splines on a FVS-triangulation. FVS-triangulation is a triangulation obtained from a quadrangulation by adding two diagonals of each quadrilateral. Such a triangulation was first considered by Fraeijs de Veubeke and Sander in 1974. They studied C^1 piecewise cubic splines on this special triangulation to construct finite elements. Their study was generalized by Laghchim-lahlou and Sablonniére in 1989. They constructed finite elements in spline spaces S_{3r}^r of smoothness r and degree $3r$ for odd integer r and in spline spaces S_{3r+1}^r of smoothness r and degree $3r+1$ for even integer r on a FVS-triangulation. In Lai 1993, an application of S_3^1 to CAGD was considered. Lai showed the dimension of S_3^1 on such a triangulation is smaller than other bivariate C^1 spline spaces. In Lai and Schumaker 1994, they studied the construction of locally supported basis and approximation order of S_6^2 on such a triangulation. They demonstrated that the dimension of this spline space is smaller than many C^2 spline spaces. In Lai 1994, Lai studied the approximation order of bivariate spline spaces $S_{d(r)}^r$ on a FVS-triangulation for $d(r) = 3r$ if r is odd and $d(r) = 3r - 1$ if $f > 2$ is even. He showed those spline spaces with lower degree $d(r) < 3r + 2$ have full approximation order.

Composite Wavelet Transforms

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Meaningful unifications of the theory of affine frames and Gabor frames are obtained by means of group extensions that contain both affine and Heisenberg subgroups. Extensions of the continuous wavelet and Gabor transforms are discussed and two methods for designing corresponding frames adaptable to analyzing specific signals are introduced.

Monotone Rational Approximation

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We consider the problem of uniform approximation on a finite set of real numbers by rational functions whose first derivative is required to be nonnegative at each of the points. We discuss computational results obtained by applying the CONMAX nonlinear optimization code (developed by the authors) to this problem. Some theorems concerning best approximations are presented, and some possible extensions of the work are mentioned.

Multivariate Polynomial Interpolation: Explicitation of Lagrange and Hermite Polynomials

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We present some explicit algorithm for the explicitation of Multivariate Hermite or Lagrange Polynomials interpolating data in $\mathbf{R}^2$ or $\mathbf{R}^d$. Techniques from real algebraic geometry are well suited for Lagrange interpolation problems in the plane, providing algorithms to decide whether a data set is unisolvent or not, and also a recursive algorithm that enables us to construct the associated Lagrange basis. For a Newton approach, it is more relevant to use a multiresolution algorithm which is available in any dimension. Finally, we consider the Abel interpolation problem in $\mathbf{R}^d$, i.e. the interpolation problem where a point A_α is associated to the derivative $\partial^\alpha f$ for any α such that $|\alpha| \leq k$. Some of the points α can coalesce, and the most obvious important case of Abel interpolation corresponds to the Taylor interpolation at a single point. The importance of these interpolation problems comes from the fact that an interpolation scheme is always regular (i.e., solvable whatever the location of the data points is) if and only if the interpolation problem is an Abel's one.

We provide an explicit formula for the solution which can be considered as a Multipoint Taylor Formula.

Approximation From Shift-Invariant Spaces by Certain Integral Operators

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We consider approximation from shift-invariant spaces by using an integral operator. We characterize the approximation order provided by such an integral operator. Applications of this characterization are also given.

On a Density Question for Neural Networks

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One of the most interesting questions connecting neural networks and approximation theory concerns the principal capability of three-layer feedforward networks to simulate a given function up to any desired accuracy. The most complete and satisfactory result for standard McCulloch-Pitts nets was recently given by Leshno et al. [2]. In this talk we consider a related question where the McCulloch-Pitts neurons are substituted by so-called hyperbolic sigma-pi neurons. We will prove that three-layer feedforward neural networks with hyperbolic sigma-pi neurons in the hidden layer can approximate any continuous multivariate function up to any desired accuracy, more precisely, that they induce a translation and dilation invariant subspace of $C(\mathbf{R}^n)$ in the topology of uniform convergence on compact subsets. For those interested in the general neural network background we refer to [1], [2], and [3]; generalizations of our results by means of completely different techniques may be found in [4].

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Generalised *sk*-spline Interpolants on Compact Abelian Groups

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Here we investigate interpolation and approximation of continuous functions on a compact Abelian group, using shifts of some fixed continuous function. We also consider some extremal properties of such interpolants. Finally, for a wide range of smooth functions on the torus, we obtain error estimates for generalised *sk*-spline interpolants.

Convex Polynomial Approximation in the L_p Norm

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If $F \in L_p[-1,1]$, $0 < p < \infty$, is convex, we would like to estimate the degree of polynomial approximation to f under the constraint that the polynomials are convex. Estimates so far involved the second modulus of smoothness of f in the L_p-norm. While it was known that estimates involving the fourth modulus of smoothness of f are impossible with an absolute constant, it was an open question whether estimates involving the third are valid. We show that for such an f and for each $n \geq 2$, there exists convex polynomials $p_n \in \Pi_n$ such that

$$\|f - p_n\|_p \leq C\omega_3^\varphi \left(f, \frac{1}{n}\right)_p$$

where $C = C(p)$ when $p \to 0$.

Distribution of Zeros and Poles of Rational Functions that are Asymtopically Optimal for the Zolotarev Problem

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Given two disjoint compact subsets E_1, E_2 of the complex plane, the generalized third Zolotarev problem concerns estimates for the quantity

$$Z_{mn} := \inf_{r \in \mathbf{R}_{mn}} Z_{mn}(r) := \inf_{r \in \mathbf{R}_{mn}} \left\{ \sum_{z \in E_1} |r(z)| \ \setminus \inf_{z \in E_2} |r(z)| \right\},$$

where $\mathbf{R}_{mn}$ denotes the class of all rational functions with numerator degree m and denominator degree n. Recently we showed that for any "ray sequence" of integers (m,n), that is $m/n \to \lambda, m+n \to \infty, Z_{mn}^{1/(m+n)}$ approaches a limit $L(\lambda)$. A sequence $\{r_{mn}\}$ is called asymptotically optimal if

$$Z_{mn}\,(r_{mn})^{1/(m+n)} \to L(\lambda).$$

Under some geometric assumptions on E_1, E_2, we describe the asymptotic behaviour of zeros and poles of such r_{mn}'s. As a consequence, we describe (for the case $\lambda \leq 1$) the behaviour of rational functions that are asymptotically optimal for the fourth Zolotarev problem (approximation of a signum-type function).

Multivariate Interpolating Wavelets and Duals

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This is joint work with C. K. Chui. We apply our general frame work of multivariate wavelets with duals to the interpolatory setting and construct multivariate interpolating wavelets as well as their distributional duals. Examples in terms of multivariate box splines are given.

Minimization of Multivariate Quadratic Splines

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We study multivariate quadratic splines that have simple matrix representations. The problem of minimizing such a spline function is closely related to quadratic programming problems, linear programming problems, Huber's M-estimator, and best ℓ_1-approximation problems. There are many different issues related to minimization of quadratic splines, including computational complexity, combinatorial analysis, algorithms, error bounds, and applications. We will give a brief overview of this interesting topic.

Laguerre Series Method for Finding the Best Hankel Approximation of Infinite Dimensional Systems

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Orthonormal Laguerre basis for $L^2(0,\infty)$ is applied to compute the best Hankel approximation of certain classes of continuous-time infinite dimensional systems in time domain. And moreover, the modulus of continuity of best Hankel approximation operator is derived.

Characterization of the Order of Polynomial-Reproduction for Multi-Scaling Functions

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If the integer translates of a compactly supported uni-scaling function φ form a Riesz basis for a subspace V_0 of $L^2(\mathbf{R})$, it is well-known that φ has polynomial-reproduction order m if and only if the two-scale symbol P of φ is divisible by the factor $(1+z)^m$ but not by $(1+z)^{m+1}$. However, this result does not extend to the multi-scaling function setting. The objective of this paper is to establish certain necessary and sufficient conditions in terms of the eigenvalues of two finite matrices for which a multi-scaling function has polynomial-reproduction order m. Some applications will be given.

Hybrid Cubic Bézier Triangle Patches on Spheres and Sphere-like Surfaces

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We discuss a new method for fitting scattered data on the sphere based on the spherical barycentric coordinates and Bernstein-Bézier polynomials introduced by Alfeld, Neamtu and Schumaker [ms, 1994]. The method extends Foley and Opitz's hybrid cubic patches [Thomas A. Foley and Karsten Opitz, Mathematical Methods in Computer Aided Geometric Design II, pp. 275–286] to spheres and general sphere-like surfaces. The hybrid form thus defined gives a local representation of cubic triangular patches, and can be evaluated with the deCasteljau algorithm. A C^1 interpolant to scattered data with cubic precision is obtained which is significantly smoother than Clough-Tocher macro elements with linear cross boundary derivatives.

Bézier Wavelets

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Classical approach to wavelets based on Mallat's multiresolution analysis for functions on the real line has been extended to functions on bounded intervals. These wavelet-like bases or multiwavelet bases for bounded intervals are much easier to construct and are not unique. A unique construction can be achieved by imposing additional requirements. For example, additional vanishing moments criteria have been used to create Legendre wavelets that have been useful in sparse representation of integral operators. More recently other wavelets have been created that do not necessarily have the translation-invariant or scale-invariant properties and have found useful applications in automatic level-of-detail control for editing and rendering curves and surfaces. Orthogonal properties of wavelets have also been given up to construct biorthogonal wavelets on bounded intervals, where the dual scaling functions are interpolating and have found applications in compressing elevation contours. Other applications of wavelets on bounded intervals include fast methods for solving problems in global illumination, image editing and compression, and surface reconstruction from contours.

There is a need to construct wavelets on bounded intervals whose coefficients reflect certain geometric properties that are inherent in scaling functions or are suitable to the applications at hand. Different basis or scaling functions however possess different geometric properties. For example, coefficients in the Bézier representation of curves carry geometric properties that allow a designer to control the shape of the curve. In contrast, coefficients in the Lagrange representation of curves carry geometric property of interpolation. Certain desirable geometric properties of wavelets from the point of view of geometry applications are symmetry, smoothness, and small support.

Here we investigate the construction of wavelets for curves represented in the Bézier bases. The de Casteljau algorithm can be viewed as the reconstruction relations for the Bézier scaling functions. In this work we study the decomposition relations for the Bézier basis functions. In particular, we construct Bézier wavelets whose coefficients carry some precise geometric information. We then discuss an application of this representation to smoothness preserving approximation of curves.

Optimal Polynomial Orthogonal Systems

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(Abstract not available)

Trigonometric Polynomial Bases of $C[0,1]$

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The problem of finding a basis of $C[0,1]$ which consists of trigonometric polynomials of lowest possible degrees has long been open. Privalov gave strict lower and upper bounds on the growth of the degrees of such a (growth-) optimal basis. More recently, Sahakian and the author used wavelet techniques to construct trigonometric bases that are not only optimal with respect to degree but are also orthogonal. This result as well as extensions will be presented.

Weighted Approximation and Orthogonal Polynomials on $(-\infty,\infty)$

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It was S. N. Bernstein who began the theory of weighted approximation on $(-\infty,\infty)$, with the following question posed about 1911: Let $W : \mathbf{R} \to [0,1]$. Under what conditions on W is it true that for every continuous $f : \mathbf{R} \to \mathbf{R}$, we can find polynomials $\{P_n\}$ with

$$\lim_{n\to\infty} \|(f - P_n)W\|_{L_\infty(\mathbf{R})} = 0.$$

This *qualitative* problem was solved by Pollard, Achieser and Mergelyan in the 1950's. It was M. Dzrbasjan who really began the *quantitative* side of the theory in the early 1950's, but he soon turned to other topics. G. Freud, assisted by P. Nevai, developed the theory in the 1970's side by side with work on orthogonal polynomials for weights such as $\exp(-|x|^\alpha), \alpha > 0$. Fundamental advances in the subject were made with the systematic use of potential theory in the 1980's until the present. We survey part of the history and the most recent results, including a selection from:

- Jackson and Bernstein Theorems
- Markov-Bernstein Inequalities
- Bounds on Orthogonal Polynomials
- Lagrange Interpolation
- Orthogonal Expansions
- A Unified Treatment of Freud, Erdös, and non-symmetric weights

A Preliminary Approach to a Meromorphic Approximation in A Strictly Levi Pseudoconvex Domain with a C^2-boundary

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Let Y be a complex manifold and suppose D is a strictly Levi pseudoconvex in Y. By a powerful classical lifting theorem, we explore the possibilities of conveying an Oka-Weil type meromorphic approximation from a Stein manifold X to D.

A Link Between Bernstein Polynomials and Durrmeyer Polynomials with Jacobi Weights

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We will study a family of linear operators P_n defined by

$$P_n(f;x) := \sum_{k=0}^{n} p_{n,k}(x) T_{\alpha,k}(f),$$

with the positive linear functionals $T_{\alpha,k} : C(I) \to \mathbf{R}$ for $k = 0, \ldots, n$

$$T_{\alpha,k}(f) := \frac{\int_0^1 f(t) t^{ck+a}(1-t)^{c(n-k)+b} dt}{B(ck+a+1, c(n-k)+b+1)}.$$

Here one used for c the special sequence in n and α, with $n \in \mathbf{N}, 0 \leq \alpha < \infty$ defined as $c := c_n := [n^\alpha]$.

This family generalizes the *classical Bernstein operators* and in terms of the modulus of continuity and the Ditzian Totik modulus of smoothness we give among other results a

- Voronovskaja Type Result,
- Global and Local Characterization Result,
- Equivalence Results and Linear Combination.

This fact of covering a great class of operators has been introduced in [6] (and was generalized in [3]). The above polynomial sequence $(P_n)_{n \in N}$ includes for example a number of interesting approximation processes: *Kantorovic polynomials* $K_n f$, the *Bernstein polynomials* $B_n f$, the *Bernstein-Durrmeyer-Lupas* polynomials $M_n f$ (J. J. Durrmeyer [2], A. Lupas [4], see also [1]), and the *de la Vallé-Poussin type polynomials* [5].

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An Efficient Lower Bound for the Generalized Spectral Radius of a Set of Matrices

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Generalized spectral radius of a set of matrices is a relatively new concept in Linear Algebra and an important tool in regularity analysis of Wavelets. Let Σ be a collection of m matrices and define $\mathcal{L}_n$ as the set of all m^n products of length n of the elements of Σ. Then for each $A \in \mathcal{L}_n$ define its *average spectral radius* by $\overline{\rho}(A) = [\rho(A)]^{1/n}$. Let $\rho_n(\Sigma) = \max_{A \in \mathcal{L}_n} \overline{\rho}(A)$ and define the generalized spectral radius of Σ as $\rho(\Sigma) = \limsup_{n \to \infty} \rho_n(\Sigma)$. The cost of calculating $\rho(\Sigma)$ generally increases exponentially with the required accuracy. Here we reduce the cost of $\rho_n(\Sigma)$ from m^n matrix calculations to m^n/n. This is accomplished by utilizing the invariance of average spectral radius under (1) cyclic permutation of elements of matrix product, and (2) exponentiation of the product. The equivalence classes introduced by the first relationship can be counted using MacMahon's formula, and within the resulting representative classes the equivalence class introduced by the second relationship can be counted using Dedekind-Liouville inversion principle. We simplify the resulting formula and establish an inequality that limits the operation cost for $\rho_n(\Sigma)$ to m^n/n.

Evaluation of Kergin Interpolants

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It is well known that the Kergin-operator $C^\mu(\mathbf{R}^r) \longrightarrow \mathbf{P}^r_\mu$ is uniquely interpolating in $\mu+1$ nodes, only. This means that it is interpolating in far less nodes than is corresponding with the dimension of $\mathbf{P}^r_\mu$.

In contrast to Kergin interpolants to the monomials, Kergin interpolants to arbitrary functions of $C^\mu(\mathbf{R}^r)$ cannot be computed directly though there exists an explicit representation of the operator given by Micchelli and Milman. The problem in their representation is that integrals of multiple directional derivatives appear.

We give another representation which is using partial derivatives, instead, and which is reducing the problem to Kergin interpolation of the monomials. Consequently, Kergin interpolation to the monomials is investigated. The coefficients of the corresponding interpolants are presented by an explicit expression by which we are able to obtain recurrence relations for them.

Using results of Micchelli concerning simplex splines, it is, after all, possible to give a computationally evaluable representation of Kergin interpolants for functions from $C^\mu(\mathbf{R}^r)$.

On the Rate of the Uniform Rational Approximation on Quasismooth Arcs

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Let L be a bounded Jordan arc in the complex plane $\mathbf{C}$. Denote by $H^\alpha(L)$, $0 < \alpha \leq 1$, the class of functions f, defined in L, which satisfy the Hölder condition

$$|f(z) - f(\zeta)| \leq c(f)|z - \zeta|^\alpha.$$

For the best uniform approximation of continuous function f on L by rational functions (polynomials) of degree at most n we use the notation $R_n(f, L)$ $(E_n(f, L))$.

If $L = [0, 1]$ is the real interval, then

$$R_n(f, L) = O\left(n^{-\alpha}\right) \tag{1}$$

for any function $f \in H^\alpha(L)$. This result follows immediately from the same estimate for $E_n(f, L)$ and can not be improved, in general, if $\alpha \neq 1$.

Now let $L = [0, 1] \cup [0, i]$. It is known that $f \in H^\alpha(L)$ implies

$$E_n(f, L) = O\left(n^{-\alpha/2}\right). \tag{2}$$

Note that the factor $1/2$ here is closely connected with the angle of L at the origin, and the estimate is precise for the class $H^\alpha(L)$. As the consequence of (2), the same relation is valid for $R_n(f, L)$, but what about the sharpness of it ?

We investigate the rate of the decrease of $R_n(f, L)$ for functions $f \in H^\alpha(L)$, $0 < \alpha \leq 1$, on quasismooth arcs (arcs without "cusps"). It turns out that the non-zero angles of L (if any) are not essential for the rational approximation unlike to polynomial one, and estimate like (1) holds for an arbitrary quasismooth arc.

Characterization of Best Approximants in L_φ Spaces

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Let $\varphi : [0, \infty) \to [0, \infty)$ be a convex function, $\varphi \not\equiv 0$, $\varphi(0) = 0$, and let $(\Omega, \mathcal{A}, \mu)$ be a measure space. Consider the set $\mathcal{L}_\varphi = \{f : \int_\Omega \varphi(\alpha|f|) < \infty \text{ for all } \alpha > 0\}$, where f is a real-(complex-)valued $\mathcal{A}$-measurable function. Then $\mathcal{L}_\varphi$ is a real-(complex-)Orlicz linear space. The classical L_p spaces, $1 \leq p < \infty$, are included in this situation. For a fixed f in $\mathcal{L}_\varphi$, consider the set $\mathcal{M}_\varphi(f)$ of elements g_0 that minimize $\int_\Omega \varphi(|f - g|)$ among the functions g of some subclass $\mathcal{M}$ of $\mathcal{L}_\varphi$. $\mathcal{M}_\varphi(f)$ is nonvoid in many contexts. If $\mathcal{M}$

is convex, then a characterization of g_0 can be obtained from the fact that the function $G : [0,1] \to [0,\infty), G(\lambda) = \int_\Omega \varphi(|f - (1-\lambda)g_0 - \lambda h|) = \int_\Omega \varphi(|f - g_0 - \lambda(h - g_0)|)$, is a bounded convex function for all h in $\mathcal{M}$. From this characterization we can get a description of g_0 for several examples of $\mathcal{M}$. In this way, known results in L_p approximation can be extended to the present L_φ approximation. Moreover, these results can now be explained in terms of the behavior of φ. Analogous conclusions are valid when approximating with the Luxemburg norm with which any Orlicz space can be endowed.

Quantizing Multiwavelet Coefficients

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We compare various quantization schemes for multiwavelet expansions using a variety of multiwavelets and wavelets.

An Efficient Variation on the Oslo Algorithm

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Algorithms for the computation of the coefficients involved in knot insertion are presented which are more robust and efficient than the Oslo algorithm, or its improvement by Lyche and Møorken. They are based on a recurrence relation which is slightly different from the one for discrete B-splines. This relation has the advantage that it is easy to determine a priori which elements in the recurrence chain are non-zero using only tests on the indices of the knots.

TO BE ANNOUNCED

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An Adapted Wavelet Packet Basis for the Parallel Solution of Image Restoration Problems

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It is well known that the image restoration problem is described by a Fredholm integral equation of the first kind and its approximate solution leads to a very large ill posed linear system. Iterative regularization methods and parallel algorithms are therefore needed [1],[2],[4]. On the contrary, a different strategy to solve such large-scale problems is based on a suitable choice of the approximation space basis. In fact, a proper choice can be used to convert the linear systems matrices to sparse form, as it happens, for example, by using orthonormal wavelet and wavelet packet bases [3],[6].

In this paper we look for a least squares solution of the restoration problem, which belongs to the uniform knots cubic spline space. We have therefore considered a multiresolution non-orthogonal wavelet packet decomposition of this space [5], which leads to an equal sized block form of the original matrix [6]. Furthermore, if the kernel of the integral equation (point spread function) is, as it usually occurs, of convolution type and with finite support, it is possible, under mild hypothesis, to construct a new compactly supported wavelet packet basis which is adapted to the considered kernel. On this basis the least squares matrix presents a nice, diagonal block form, which is a good starting point for the realization of parallel algorithms well suited for coarse grained distributed memory parallel machines.

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L^∞ Markov-Bernstein Inequalities for Erdös Weights

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Let $W(x) := \exp(-Q(x)), x \in \Re$, where $Q(x)$ is even, sufficiently smooth, and is of faster than polynomial growth at infinity. Let a_n denote the n^{th} *Mhaskar–Rahmanov–Saff* number for Q, and $\delta_n := (nT(z_n))^{\frac{2}{3}}, n \geq 1$, where T is an increasing function. For $|x| \leq a_n(1 + 2L\delta_n)$, we define

$$\Psi_n(x) := \max\left\{\sqrt{\frac{1-|x|}{a_n} + 2L\delta_n}, \frac{1}{\left[T(z_n)\sqrt{\frac{1-|x|}{a_n+2L\delta_n}}\right]}\right\}.$$

We prove that for $n \geq 1$, and polynomials P of degree at most n, there is a $C > 0$ such that

$$\|(PW)'\Psi_n(x)\|_{L^\infty(\Re)} \leq \frac{C_n}{a_n}\|PW\|_{L^\infty(\Re)}.$$

Climate Surfaces for Texas Using Thin Plate Splines

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and Texas A&M University
Temple, Texas and Canberra

Thin plate splines and the digital elevation model (DEM) are used to fit monthly average precipitation and temperature surfaces for Texas. The elevation of the gauging station is used as an auxiliary variable for interpolation of the precipitation surfaces. The surfaces compare very favorably with existing climate contour maps. When surfaces are fit for precipitation without the DEM, large residual errors result. The ANUSPLIN software that was used is easily learnt by a novice, allows a user to easily locate stations that have problems in their datasets, and runs within a few minutes for a grid of 1000×1000 cells of Texas. This makes it particularly attractive for Geographic Information Systems (GIS). The effects of removing observations from input to the spline model are explored.

Nonstationary Spherical Wavelets for Scattered Data

F. J. Narcowich* and J. D. Ward
Texas A&M University
College Station, Texas

We construct a class of nonstationary, orthogonal, wavelets generated by what we call *spherical basis functions* (SBFs), which comprise a subclass of Schoenberg's positive definite functions on the m-sphere. We will discuss decomposition, reconstruction, localization, and algorithms for analyzing scattered data on the m-sphere.

Reproducing Kernel Hilbert Spaces in Computational Harmonic Analysis

M. Z. Nashed
University of Delaware
Newark, New Jersey

Reproducing kernel Hilbert spaces (RKHS) play an important role in several areas of approximation theory, numerical analysis, and estimation theory. In recent years, the use of RKHS has led to interesting interactions between harmonic analysis, signal processing, and computational mathematics. In this talk, I will indicate some results of the interaction, with emphasis on sampling expansions, integral equations, and numerical conformal mappings.

Sparse Approximate Multiquadric Interpolation

B. K. Natarajan* and R. E. Carlson
Hewlett Packard Laboratories and Lawrence Livermore Labs

Multiquadric interpolation is a technique for interpolating scattered samples of multivariate functions, typically to support data visualization. We present an algorithm for computing sparse but approximate multiquadric interpolants. Such interpolants are useful since (1) the cost of evaluating the interpolant scales directly with the number of coefficients, and (2) the principle of Occam's Razor suggests that the interpolant with fewer coefficients better approximates the underlying function.

Spherical Splines

P. Alfeld, M. Neamtu*, and L. Schumaker

Vanderbilt University
Nashville, Tennessee

We present a natural way to define spaces of splines on the sphere and on sphere-like surfaces, which is based on a theory of spherical Bernstein-Bézier polynomials. Such spaces exhibit most of the important properties of the classical BB-polynomials on planar triangulations. The methods discussed have immediate applications to a variety of important practical problems including interpolation, data fitting, and modelling of surfaces defined on the sphere.

Perturbation of Orthogonal Polynomials on An Arc of the Unit Circle

Paul Nevai* and Walter Van Assche

Ohio State University, Columbus, Ohio and
Katholieke Universiteit Leuven, Leuven, Belgium

Orthogonal polynomials on the unit circle are completely determined by their reflection coefficients through the Szegö recurrences. We assume that the reflection coefficients converge to some complex number a with $0 < |a| < 1$. The polynomials then live essentially on the arc $\{e^{i\vartheta} : \alpha \leq \vartheta \leq 2\pi - \alpha\}$ where $\cos\frac{\alpha}{2} \stackrel{def}{=} \sqrt{1-|a|^2}$ with $\alpha \in (0,\pi)$. We analyze the orthogonal polynomials by comparing them with the orthogonal polynomials with constant reflection coefficients, which were studied earlier by Ya. L. Geronimus and N. I. Akhiezer. In particular, we show that under certain assumptions on the rate of convergence of the reflection coefficients the orthogonality measure will be absolutely continuous on the arc with a continuous weight function.

Non-Stationary Wavelet Bases for Sobolev Spaces

Igor Ya. Novikov

Voronezh State University
Voronezh, Russia

An approach for the construction of non-stationary orthonormal wavelets is presented. The method allows us to obtain non-stationary orthonormal infinitely differentiable compactly supported wavelets. It is proved that these wavelets form a Riesz basis for all Sobolev spaces simultaneously. The estimates of the uncertainty bounds of non-stationary wavelets will be discussed.

On the Peculiarities of Canonical Injection Operators for Rearrangement Invariant Spaces

S. Ya. Novikov
Samara State University
Samara, Russia

(Abstract not avaialble)

Bivariate Spline Interpolation and Approximation Order

Günther Nürnberger
University of Mannheim
Mannheim, Germany

We describe a general method for constructing point sets which admit unique Lagrange and Hermite interpolation by spaces of bivariate splines of arbitrary degree and smoothness. The splines are defined on uniform type triangulations of simply connected domains. It is shown that Hermite interpolation by splines of smoothness one yields (nearly) optimal approximation order.

Numerical examples using 2000 Lagrange interpolation points are given. For computing the interpolating splines, only small systems have to be solved. Interpolation methods for spline spaces of special degree are known in the literature.

On the Number of Orthogonal Wavelet Multiplier Matrices

Gerhard Opfer
University of Hamburg
Hamburg, Germany

Scaling equations have usually the form

$$\varphi(\mathbf{x}) = \sum_{\mathbf{k}\in\mathbf{Z}^n} c_{\mathbf{k}}\varphi(\mathbf{A}\mathbf{x}-\mathbf{k})$$

where $\mathbf{A}$ is a square, nonsingular matrix of order n with integer entries and therefore, $\det(\mathbf{A})$ is a non vanishing integer. In addition, it is required that $\mathbf{A}$ has a certain stretching property expressed by $|\lambda(\mathbf{A})| > 1$ which should mean that all eigenvalues of $\mathbf{A}$ are located outside the closed unit disk. Since the determinant is the product of the eigenvalues, it follows that $|\det(\mathbf{A})| \geq 2$. In the applications the matrices considered are orthogonal in the sense that $\mathbf{A}\mathbf{A}^{\mathrm{T}} = c\mathbf{I}$ where $\mathbf{I}$ is the identity matrix and c a certain constant. One is usually interested in matrices $\mathbf{A}$ where the absolute value of the determinant is small, because $|\det(\mathbf{A})| - 1$ is the number of wavelets necessary to span $\mathbf{L}_2(\mathbf{R}^n)$. Our lecture

will mainly be concerned with the number of matrices with the prescribed properties when either $|\det(\mathbf{A})|$ is prescribed or is bounded by a given constant. There will be results for general n and some explicit counting results for $n = 3$ and $n = 4$.

Piecewise Linear Prewavelets of Small Support

Uwe Kotycka and Peter Oswald*
Friedrich-Schiller Universität Jena
Jena, Germany

For preconditioning elliptic problems in Sobolev spaces H^s over domains, multilevel frames composed of finite element nodal basis functions have gained popularity over the last five years. For certain applications, the construction of L_2-semiorthogonal prewavelets related to these nodal basis frames is desirable. We give first results in the special case of piecewise linear finite element functions on sequences of triangulations in the two-dimensional case. One topic considered is the minimization of the size of the underlying "refinement" masks.

On Continuity of the Prox Map and the Restricted Center Map

D. V. Pai* and B. M. Deshpande
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Powai, Bombay, INDIA

In this paper we review some of the recently introduced hypertopologies on collections of nonempty closed subsets of a metrizable space in the context of continuity properties of the prox map and the restricted center map. We identify here suitable families $\mathcal{A}, \mathcal{B}$ of nonempty closed sets equipped with natural topologies, as coordinate spaces for the gap functional and the restricted radius functional, with a view to explore bivariate continuity of these functionals. This leads us to sharpenings of many known results as well as to some new results for upper semicontinuity of the prox map and the restricted center map. This, in turn, also leads us to improvements of certain best approximation results for convex-valued multifunctions and fixed point theorems for such multifunctions.

An Equivalence Relation Between Multiresolution Analyses of $L^2(\mathbf{R})$

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Hellenic Military Academy
Athens, Hellas (Greece)

Classification problems are of major importance in various fields of mathematics. We

present a natural notion of equivalence called unitary equivalence between multiresolution analyses of $L^2(\mathbf{R})$.

Let $\mathcal{M} = \{V_j : j \in \mathbf{Z}\}$ be a multiresolution analysis of $L^2(\mathbf{R})$ (MRA) and φ in $L^2(\mathbf{R})$. We say that φ is a *scalingfunction* for $\mathcal{M}$ if $\{\varphi_{0,n} : n \in \mathbf{Z}\}$ is an orthonormal basis of V_0.

DEFINITION 1. *Two MRAs $\mathcal{M}$ and $\mathcal{N}$ on $L^2(\mathbf{R})$ are called unitarily equivalent if there exist scaling functions φ and χ for $\mathcal{M}$ and $\mathcal{N}$ respectively and a unitary operator U defined on $L^2(\mathbf{R})$ such that $U\varphi_{m,n} = \chi_{m,n}$, $m, n \in \mathbf{Z}$.*

Employing operator theoretic techiques, we can prove the following characterization of the unitary equivalence.

THEOREM 2. *Let $\mathcal{M}$ and $\mathcal{N}$ be two MRAs. Then they are unitarily equivalent iff there exist scaling functions φ for $\mathcal{M}$ and χ for $\mathcal{N}$ such that $\hat{\chi}(t) = h(t)\hat{\varphi}(t)$ a. e. , where h is a Borel measurable function defined on $\mathbf{R}$ such that $|h(t)| = 1$ and $h(2t) = h(t)$ a. e.*

The equivalence relation we introduced is naturally associated with the well-known Decomposition and Reconstruction algorithms of Mallat defined for every MRA.

THEOREM 3. *Let $\mathcal{M}$ and $\mathcal{N}$ be two MRAs of $L^2(\mathbf{R})$. Then $\mathcal{M}$ and $\mathcal{N}$ are unitarily equivalent iff they give the same Decomposition and Reconstruction algorithms.*

Theorem 3 allows us to obtain a partial generalisation of the uniqueness results of I. Daubechies and J. C. Lagarias concerning the solutions of two-scale difference equations. Theorem 2 provides a tool to examine whether any two given MRAs are unitarily equivalent. The conditions for the function h of Theorem 2 seem rather restrictive. There are certain cases where the equivalence classes are singletons. Focusing to the functions h which are constant in each of the intervals $(-\infty, 0)$ and $[0, +\infty)$, we obtain equivalences implemented by certain linear combinations of the identity operator and the Hilbert Transform. Finally we prove that MRAs defined by B-splines of different degree are non-equivalent.

On the Best Approximation of Abstract Integrable Functions

Irina Peterburgsky
Suffolk University
Boston, Massachusetts

Let E be a conjugate complex Banach space, L be a Banach space of Bochner integrable functions from the unit circumference of a complex plane to E, and K be a subspace of L that consists of functions with zero Fourier coefficients of non-positive indices. For any g belonging to L, a problem about

$$\inf_h \|g - h\|_L, \quad h \text{ from } K, \tag{1}$$

is considered. The existence of the best approximation in (1) has been established based on specific properties of Banach-space-valued functions obtained in the paper.

Polynomial Interpolation and Peano Kernels

Knut Petras

Institut für Angewandte Mathematik
Braunschweig, Germany

A description of the Peano kernel method for the estimation of errors of polynomial interpolation is given. The dependence of the error bounds, which use the first derivative, on the location of the evaluation point is characterized. The results are applied to polynomial interpolation involving Chebyshev nodes. Sharp bounds for the error estimation with the first derivative as well as asymptotic estimates involving higher derivatives are derived.

Similar results as in the algebraic case are also proved for trigonometric interpolation involving equidistant nodes.

Approximation Properties of Neural Networks

Ron DeVore, Konstantin Oskolkov, Pencho Petrushev*, and Vladimir Temlyakov

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Columbia, South Carolina

P. Petrushev
Bulgarian Academy of Sciences
Sofia, Bulgaria

Neural networks are being proposed as an efficient approximation method in several applications. However, their approximation properties are not well understood. This talk will present recent results on the degree of approximation possible from linear and nonlinear spaces generated by sigmoidal functions and other ridge functions. In particular, we shall show that it is possible to obtain optimal approximation order for classical function spaces from certain linear spaces generated by sigmoidal functions.

Nonorthogonal Compactly Supported Wavelets

Alexander P. Petukhov

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We consider here only the 1-periodic case, but most of the facts are valid for the nonperiodic one. Let $\mathbf{T} = [0, 1]$, the sequence of spaces $V_0 \subset V_1 \cdots \subset V_j \subset \ldots$ generate a Multiresolution approximation (MRA) of $\mathbf{L}^2(\mathbf{T})$, the corresponding translates of the function φ^j form a basis (in general nonorthogonal) of $V_j, V_{j+1} = W_0 \oplus W_1 \oplus \cdots \oplus W_j, j =$

$0, 1, 2, \ldots$, be an orthogonal sum of wavelet spaces,

$$\alpha_i = \int_0^1 \varphi^j(x) \cdot \overline{\varphi}^{j+1}(x - i/2N)dx, i = 0, 1, \ldots, 2N-1, \text{ where } N = 2^j.$$

PROPOSITION. *Suppose*

$$\psi^j(x + 1/2N) = \sum_{n=0}^{2N-1} (-1)^n \overline{\alpha}_n \varphi^{j+1}(x + n/2N).$$

Then the system $\{\psi^j(x - k/N)\}_{k=0}^{N-1}$ *of functions forms a basis of the space* W^j.

In particular, it follows from the Proposition that if supp $\varphi^j < p/N$ for all j (it corresponds to the case of a finite support for $\mathbf{L}^2\mathbf{R}$)), then supp $\psi^j < 2p/N$. Hence, it is easy to construct algorithms of decomposition and reconstruction for elements of V^j which have the complexity $O(pN)$ (the same as for the case of an orthonormal basis).

Unfortunately, the mentioned algorithm has dealt with solving of systems of linear equations, so it cannot be used in the real time mode. However, if we do not require orthogonality of the spaces W^j and we only suppose that $V_{j+1} = W_0 + W_1 + \cdots + W_j$, where the sum is direct, then there exists a decomposition of V_j of such type for which a decomposition algorithm can be realized in real time as a discrete convolution with a finite window. If at the same time two-scale relations between the spaces V_j take place, then an algorithm of reconstruction can be also realized a discrete convolution with a finite window.

Besides, we establish a natural one-to-one correspondence between all possible MRA of $\mathbf{L}^2(\mathbf{T})$ and functions $\mu \in \mathbf{L}^2(\mathbf{T})$ for which $\hat{\mu}(2k+1) \neq 0$ for any $k \in \mathbf{Z}$.

Perturbation of Orthogonal Polynomials On an Arc of the Unit Circle

Leonid Golinskii, Paul Nevai, Walter Van Assche, and Frank Pinter*

Ohio State University
Columbus, Ohio

Katholieke Universiteit Leuven
Leuven, Belgium

The perturbation analysis of orthogonal polynomials on an arc of the unit circle was initiated by L. Golinskii, P. Nevai, and W. Van Assche, and, simultaneously by F. Peherstorfer and R. Steinbauer. They utilized the one-step matrix recurrence relation to derive asymptotic results about the perturbed system of orthogonal polynomials and to describe the new measure of orthogonality.

This is a joint work with L. Golinskii, P. Nevai, and W. Van Assche. The purpose of this talk is to obtain further asymptotic results and conclusions about the measure using

different techniques. We use the less obvious three-term recurrences along with former techniques developed by P. Nevai, T. S. Chihara and A. Máté for orthogonal polynomials on the real line; as well as incorporate the decomposition of linear recurrences to place our results into much more general context.

Real and Complex Interpolation of Operator Spaces

Gilles Pisier

Texas A&M University, College Station, Texas, and
Université Paris, Paris France

The talk will discuss several results on the real and complex interpolation method applied to couples of operator spaces of the form $(B(X_0), B(X_1))$, where (X_0, X_1) is a couple of Banach spaces and $B(X)$ denotes the space of all bounded linear operators on X. The main example is the case $X_0 = L_\infty$, $X_1 = L_1$. In that case we are able to describe the real and complex interpolation spaces and to compute an equivalent of the K_t-functional of Peetre. In the last quarter of the talk we will discuss the non-commutative case, *i.e.* the case when X_0 (resp. X_1) is a "non-commutative" L_∞-space (resp. L_1-space), *i.e.* a von Neumann algebra (resp. the predual of a von Neumann algebra).

Multiwavelets With Short Support

Gerlind Plonka

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We want to generalize the concept of cardinal wavelets in the following way: We introduce the shift—invariant sample samples V_j of level j spanned by a finite set of linearly independent (orthogonal) scaling functions φ_k $(k = 0, \dots, r-1)$

$$V_j := \operatorname{span}\{\varphi_k(2^j \cdot -\ell);\ k = 0, \dots, r-1,\ \ell \in \mathbf{Z}\}$$

with $r \in \mathbf{N}$.

The necessary and sufficient conditions for obtaining a generalized multiresolution analysis $\{V_j\}_{j \in \mathbf{Z}}$ will be analyzed. In particular, a matrix refinement equation for the scaling functions φ_k $(k = 0, \dots, r-1)$ must be satisfied. It is possible to construct scaling functions that have extra properties which can not be achieved with only one scaling function (cf. Daubechies). The new scaling functions can be required to be symmetric (linear phase) and with short support while their translates form an orthogonal family. Furthermore, we want to obtain a certain approximation order p of V_j. Some results by Donovan–Geronimo–Hardin–Massopust and Strang–Strela for the case $r = 2$ can be generalized.

Then our next step is to construct associated wavelets ψ_k $(k = 0, \ldots, r-1)$ by finding an orthonormal basis of the shift-invariant wavelet space

$$W_j := V_{j+1} \ominus V_j$$

in the form

$$W_j = \operatorname{span}\{\psi_k(2^j \cdot -\ell) : k = 0, \ldots, r-1,\ \ell \in \mathbf{Z}\}.$$

Our main goal is the construction of new wavelets with shortest support. The very short support of wavelets opens new possibilities for applications, for instance in numerical solution of operator equations.

Interpolation and Incidence Geometry

Burkard Polster

University of Canterbury
Christchurch, New Zealand

Around 1950 people on both sides of the pure/applied divide in mathematics started to work on very similar problems. On the one hand, people interested in interpolation theory, approximation, and convexity started to explore non-linear systems of interpolating functions. On the other hand, incidence geometers started to investigate topological affine planes, projective planes, Laguerre planes, etc. Over the past forty years both areas have been developed considerably, though there has been little or no communication between the two groups.

I will describe some of the fundamental objects of interest to both groups from both possible points of view. I will also talk about some recent progress that has been made by bringing together results from both areas.

Fast Poisson Solvers in Shift-Invariant Spaces

Gisela Pöplau

University of Rostock
Rostock, Germany

Using periodic shift-invariant spaces, we present a new approach to fast Poisson solvers. Our consideration generalizes the known concept of fast Poisson solvers. A convenient tool to construct new fast Poisson solvers is given. This method is demonstrated in periodic shift-invariant spaces spanned by translates of box splines.

Further, we obtain error estimates for the approximate solution of the Poisson equation in periodic shift-invariant spaces. Strang-Fix conditions are essential for these error estimates. The close relation to interpolation in periodic shift-invariant spaces is investigated.

The Fourier technique provides very efficient numerical algorithms based on fast Fourier transforms. Numerical tests using box splines will be presented.

Polynomial Bases for Spaces of Continuous Functions

Jürgen Prestin
Universität Rostock
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It is the aim of the talk to investigate orthogonal polynomial bases for the spaces $C_{2\pi}$ or $C[-1,1]$. Of special interest are bases $\{p_n\}$ where p_n is a polynomial of small degree. Recently, R. Lorentz and A. A. Sahakian constructed such an orthogonal basis consisting of trigonometric polynomials of optimal degree.

Here we consider finite dimensional nested spaces of trigonometric polynomials constructed from de la Vallée Poussin means of the Dirichlet kernel. Following an approach of A. A. Privalov (1991) we investigate the corresponding Multiresolution Analysis. The scaling functions and wavelets are given explicitly as trigonometric fundamental interpolants and decomposition and reconstruction algorithms can be described in simple matrix notation. The circulant structure of all relevant matrices allows the use of Fast-Fourier-Transform techniques for the actual implementation. Thus we achieve almost optimal complexity compared to other wavelet approaches derived from implicit two-scale relations, while dealing with a fully computable trigonometric multiresolution analysis with explicit algebraic formulas.

Furthermore we describe corresponding wavelet packets which yield refined frequency localization properties and can be used for the direct construction of certain orthogonal polynomial bases of $C_{2\pi}$ with optimal degree. For the corresponding partial sum operators we obtain estimates with explicit constants.

The special structure of the underlying de la Vallée Poussin means allows to transform most of these results to the algebraic case. In particular we obtain algebraic polynomial wavelet bases for $C[-1,1]$, where certain interpolation conditions for the wavelets are satisfied. Here the orthogonality is with respect to the Chebyshev weight.

Extensions of a Theorem of Ky Fan

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Let M be a compact and convex non-empty subset of a normed space E. Let f, g and h be three functions from M into E such that $\|g(x) - f(x)\| \leq \|h(x) - f(x)\|$ for all $x \in M$. We prove that when f and g are continuous and h is almost quasi-convex, there exists $x_0 \in M$ such that $\|g(x_0) - f(x_0)\| \leq \|z - f(x_0)\|$ for all $z \in \overline{S}(h(M); g(x_0))$, where for any set $X \subset E$ and $v \in E, S(M; v)$ is the set of all $z \in E$ of the form $z = \lambda t + (1 - \lambda)v$ for $t \in X$ and $\lambda \geq 1$. The same result holds for the weak topology if we assume $f(M)$ relatively compact. As corollaries we get coincidence and fixed point results, some of them from the literature and several new ones. The main tool used is a lemma due to Ky Fan.

Recognition of Biological Shapes by Means of A Wavelet Analysis

M. Cotronei and L. Puccio*
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Most of the research efforts in quantitative morphology deals with shape determination of the objects (e.g., biomedical forms) under investigation. Recently, we have faced this kind of problem with an analytical approach using an algorithm based on spline approximation. In this work we present a method for the assessment of form descriptors by means of wavelet and wavelet-packet analysis. We use a multivariate unsupervised learning scheme (cluster analysis) as objective estimator of the recognition process. The classifier objects, taken as input data, are some fundamental geometrical shapes.

Trigonometric Wavelets for Hermite Interpolation

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Center for Approximation Theory
Texas A&M University
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In this talk, a multiresolution analysis of nested subspaces of trigonometric polynomials is investigated. The pair of scaling functions which span the sample spaces are fundamental functions for Hermite interpolation on a dyadic partition of nodes on the interval $[0, 2\pi)$. Two wavelet functions that generate the corresponding orthogonal complementary subspaces are constructed so as to possess the same fundamental interpolatory properties as the scaling functions. Together with the corresponding dual functions, the

interpolatory properties of the scaling functions and wavelets are used to find the specific decomposition and reconstruction sequences of this trigonometric multiresolution analysis.

A Degree Estimate for Curvature Continuous Subdivision Surfaces

Ulrich Reif
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Stuttgart, Germany

Subdivision schemes like the Catmull-Clark algorithm admit the generation of smooth surfaces from control meshes of arbitrary topological type. Recently, this more or less empirical fact was verified by the speaker. A short survey of these results providing sufficient conditions for convergence to tangent plane continuous limit surfaces is given. When seeking algorithms converging to curvature continuous surfaces, which are unknown so far, the following estimate has to be taken into account: Consider an algorithm generating limit surfaces which consist of piecewise polynomial patches of bi-degree d joining parametrically smooth of order k. Further, assume that the algorithm is cyclic, i.e., all sides of an n-sided configuration are treated equally, and that it is not degenerated, i.e., there are no enforced flat spots. Then the limit surface can be curvature continuous only if $d \geq 2k+2$. In particular, the minimal degree is $d_{\min} = 2k_{\min} + 2 = 6$. This result applies even to non-stationary and non-compactly supported schemes.

Piecewise Convex/Concave Function Estimation and Model Selection

Kurt S. Riedel
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We consider estimation problems where the function is known to consist of a small number of pieces on which it is strictly concave or convex. To estimate the function we need to identify these regions and perform a constrained fit. The function estimation problem consists of three steps: 1) Given the data, fit the best piecewise convex function to the data; 2) Given the number of intervals, estimate the change point locations; and 3) Determine the number of change point locations. We refer to the last step as the model selection problem.

We consider a function $g(t)$ measured at N locations $\{t_i, i = 1 \ldots N\}$. The measurements are noisy: $y_i = g(t_i) + \epsilon_i$ where the ϵ_i are independent random variables with variance $[\epsilon_i] = \sigma^2$. Initially, we seek an estimate only at the measurement points: $\hat{g}_i = \widehat{g(t_i)}$. If $g_i(t)$ is convex, it satisfies the constraints

$$a_i g(t_{i-1}) + (a_i + a_{i-1}) g(t_i) + a_{i-1} g(t_{i+1}) \geq 0 \tag{1}$$

where $a_i \equiv t_{i+1} - t_0$. We begin by fitting $g(t)$ assuming that $g(t)$ is convex. This gives $N-2$ linear constraints on the least squares problem:

$$\text{Min} \parallel \mathbf{y} - \hat{\mathbf{g}} \parallel^2 \text{ subject to } \mathbf{Ag} \geq 0 \tag{2}$$

where $\mathbf{A}$ is the tridiagonal matrix with entries a_i. We denote the mean square error of the constrained fit by σ_0^2. We then try all possible fits with one change from convex to concave.

In general, we allow K changes and consider $\binom{N-2}{K}$ choices of possible change points. We denote the candidate set of change points by $S_1(w) \ldots S_k(w)$. We then minimize the least squares error subject to the constraints

$$-1^{\gamma_i(w)}[a_i\hat{g}_{i-1} + (a_i + a_{i-1})\hat{g}_i + ai - 1\hat{g}_{i+1}] \geq 0 \ ,$$

so where $\gamma_i(w) = 1$ if $S_{2\ell}(w) \leq t_i \leq S_{2\ell+1}$ and $\gamma_i(w) = 0$ if $S_{2\ell+1}(w) \leq t_i \leq S_{2\ell+2}$ for each set of change points. This is a quadratic programming problem and there is an unique solution. The minimization problem may be solved exactly using "active set" methods in quadratic programming or may be solved approximately using projected successive over relaxation (PSOR) iterations. The PSOR method uses the band structure in $A(w)$ while the active set programs require modification to take advantage of the tridiagonal structure. In both cases, the following dual formulation is easier to implement:

$$\lambda_0 = \text{argmin}_\lambda \parallel \mathbf{A}_\omega^T\lambda + y \parallel^2 \quad \text{subject to } \lambda \geq 0$$

where $\mathbf{A}_\omega$ is the $(N-2) \times N$ constraint matrix whose i-th row is given by (2). The principal advantage of the dual formulation is that the positivity constraints are easier to implement than the three point difference inequality constraints of (2). In principle, all $2\binom{N-2}{K}$ cases need to be evaluated to find the best fit. Since this is computationally costly, we consider suboptimal methods which examine only a subset of the possible change points. These candidate change points are determined by 1) initializing with a kernel smoother, 2) examining long runs of data points where the constraints are simultaneously active.

Given the best fit with K changes from convex to concave and mean square error σ_k^2 and the best fit with $K+1$ change points, we need to decide which model is preferable. We examine a variety of different model selection criteria including information criteria and Bayesian criteria.

Smooth Refinable Functions Provide Good Approximation Orders

Amos Ron

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We apply the general theory of approximation orders of principal shift-invariant spaces of de Boor, DeVore, Ron, TAMS, 1994, to the special case when the generator

ϕ of the underlying space satisfies an n-scale relation (i.e., is a "father wavelet"). We show that the approximation orders provided by such principal shift-invariant spaces are bounded from below by the smoothness class of the scaling function, as well as by the decay rate of its Fourier transform. Specifically, it is shown that, under some mild technical conditions on the scaling function ϕ, approximation order k is provided if either ϕ lies in the Sobolev space W_2^{k-1}, or its Fourier transform $\hat{\phi}(w)$ decays near ∞ like $o(|w|^{k-1})$.

The first class of results (those that are concerned with the condition $\phi \in W_2^{k-1}$) improve previously known results of Yves Meyer and of Cavareta-Dahmen-Micchelli.

The techniques apply to non-stationary ladders as well, as well as to ladders generated by several functions.

If you are aware of further known results on this topic, please share this information with the author.

Rational Approximation with Varying Weights I

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We investigate two problems concerning uniform approximation by weighted rationals $\{w^n r_n\}_{n=1}^{\infty}$, where $r_n = p_n/q_n$ is a rational function of order n. Namely, for $w(x) := e^x$ we prove that uniform convergence to 1 of $w^n r_n$ is not possible on any interval $[0, a]$ with $a > 2\pi$. For $w(x) := x^\theta$, $\theta > 1$, we show that uniform convergence to 1 of $w^n r_n$ is not possible on any interval $[b, 1]$ with $b < \tan^4(\pi(\theta - 1)/4\theta)$. (The latter result can be interpreted as a rational analogue of results concerning "incomplete polynomials".) More generally, for $\alpha \geq 0, \beta \geq 0, \alpha + \beta > 0$, we investigate for $w(x) = e^x$ and $w(x) = x^\theta$, the possibility of approximation by $\{w^n p_n/q_n\}_{n=1}^{\infty}$, where $\deg p_n \leq \alpha n, \deg q_n \leq \beta n$. The analysis utilizes potential theoretic methods. These are essentially sharp results though this will not be established in this paper.

On A Method for the Numerical Solution of Singular Integral Equations of Cauchy Type

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It is well-known that the Cauchy Singular Integral Equations

$$a(x)v(x) + \frac{b(x)}{\pi}\int_{-1}^{1}\frac{v(t)}{t-x}dt + \int_{-1}^{1}k(x,t)v(t)dt = f(x), x \in (-1,1), \tag{1}$$

are involved in several problems of a wide variety of mathematical, physical, and engineering areas.

In this paper, we consider the problem of numerically solving singular integral equations of type (1), with a and b constant.

The approach is based on a discretization of the problem, obtained by the use of suitable quadrature rules, related to weight functions of Jacobi type.

The existence and uniqueness of the solution of the numerical problem are proved; these results are accomplished by obtaining the expression of the inverse of a certain matrix related to the above problem.

A Newton Method for Polynomial Interpolation of Minimal Degree

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Let $x_1, \ldots, x_N$ be a sequence of points in $\mathbf{R}^d$. The task of polynomial interpolation of minimal degree is to find a minimal n such that there exists a subspace P of Π_n^d, the space of polynomials in d variables of total degree at most n, where the Lagrange interpolation problem is uniquely solvable; i.e., for any $f : \mathbf{R}^d \to \mathbf{R}$ there exists a unique $p \in P$ such that

$$f(x_i) = p(x_i), \qquad i = 1, \ldots, N.$$

A particular example of minimal degree interpolation is the concept of least interpolation, introduced by de Boor and Ron.

The purpose of the talk is to present a method to construct the space P and derive a Newton method based on a finite difference which also leads to a remainder formula of interpolation in terms of directional derivatives and simplex splines.

Optimal Error Bounds for Spline Interpolation on a Uniform Partition

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An n-degree spline s defined over the uniform partition, $\pi_h^a = a + \mathbf{Z}h$, of the real line $\mathbf{R}$ of mesh size h is a function $s \in C^{n-1}(\mathbf{R})$ such that s restricted to $[a + lh, a + (l+1)h]$ is an algebraic polynomial of degree at most n for any $l \in \mathbf{Z}$.

To simplify we use the notation $x_{l+t} = a + (l+t)h$. For a function f defined on $\mathbf{R}$ and $t \in \mathbf{R}$, we write $f_{l+t} = f(x_{l+t})$. A spline s is said to be the interpolating spline of f if,

for a given $v \in [0,1]$, we have $s_{l+v} = f_{l+v}$ for all $l \in \mathbf{Z}$. Also, $e(x) = f(x) - s(x)$ is called the error function.

Under some conditions on f and v, we show the two following inequalities.

$$|e^{(k)}_{l+u}| \leq A^k_n(u,v) h^{n+1-k} \|f^{(n+1)}\|_\infty \tag{1}$$

and

$$\|e^{(k)}\|_\infty \leq A^k_n(v) h^{n+1-k} \|f^{(n+1)}\|_\infty \tag{2}$$

for $k = 0, \ldots, n$, where

$$A^k_n(u,v) = \int_{-\infty}^{\infty} |N^k_n(u,v,\vartheta)|\, d\vartheta \quad \text{and} \quad A^k_n(v) = \sup_{u \in [0,1]} A^k_n(u,v)$$

are optimal bounds, and $N^k_n(u,v,\vartheta)$ is a Peano kernel.

In a previous work, we have studied inequalities similar to (1) and (2), but the optimality of the explicit bounds obtained was only proved for $k = 0$. Since $A^0_n(v)$ has a minimum at $v = v_n$ where $v_n = 0$ if n is odd, and $v_n = \frac{1}{2}$ if n is even, we only consider the case where $v = v_n$.

Our main goal is to develop useful techniques to obtain the numerical values $A^k_n(u, v_n)$ and $A^k_n(v_n)$. Using Euler-Frobenius polynomials, we obtain new representations for the kernels $N^k_n(u,v,\vartheta)$. Using a Budan-Fourier Theorem for HB-splines, it follows that $N^k_n(u, v_n, \vartheta)$ is an HB-spline having a root at each integer (usually simple) and possibly another one called $\psi(u)$. When $\psi(u)$ does not exists, we obtain that all roots are simple and deduce the value of $A^k_n(u, v_n)$. Those values are related to Euler splines. When $\psi(u)$ exists, the problem is much more difficult. We obtain results for $n = 2$ and $n = 3$ in that case.

Multivariate Interpolation and Approximation

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This survey talk will touch the following topics:

Short introduction into the theory of multivariate interpolation and approximation by finitely many (irregular) translates of a (not necessarily radial) *basis function.*

Native spaces of functions associated to conditionally positive definite functions, and relations between such spaces.

Operators on radial functions and construction of *compactly supported* positive definite radial functions.

Error bounds and *condition numbers* for interpolation.

Uncertainty Relation: Why are good error bounds always tied to bad condition numbers?

Optimality questions for approximation orders: Quasi–Optimality, Superconvergence, and Saturation.

Approximation on SO(3) and S^3 by Splines and Wavelets

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The compact 3-dimensional Lie group SO(3) of all orthogonal 3×3 matrices with determinant $+1$ which describe the rotations in $\mathbf{R}^3$ appears in many very different real world applications like robotics or molecular design. It can be parameterized by only one chart (C, τ) with $C := [-\pi, \pi] \times [-\pi, \pi] \times [-\pi/2, \pi/2]$ using Eulerian angles $\tau : (\varphi, \psi, \vartheta) \in C \longrightarrow \mathrm{SO}(3)$. The coordinate representation $f = F \circ \tau : C \longrightarrow \mathbf{R}$ of a function $F : \mathrm{SO}(3) \longrightarrow \mathbf{R}$ on SO(3) has to fulfill certain obvious periodicity conditions on the boundary of C. For $\vartheta = \pm\pi$ additional conditions apply due to the singular behaviour of τ. Furthermore for differentiable functions F on SO(3) we have the following characterization:

THEOREM. *F is C^1 if and only if f fulfills the periodicity conditions and there are real-valued $2\pi-$periodic C^1 functions $F_\pm$ as well as 2π-periodic continuous functions $A_\pm, B_\pm$ such that for $-\pi \le \varphi \le \pi$ and $-\pi \le \psi \le \pi$:*

$$f(\varphi, \psi, \pm\pi/2) = F_\pm(\varphi \pm \psi) \tag{1}$$

$$f_\vartheta(\varphi, \psi, \pm\pi/2) = A_\pm(\varphi \pm \psi)\cos\varphi + B_\pm(\varphi \pm \psi)\sin\varphi, \tag{2}$$

$$(f_\varphi \mp f_\psi)_\vartheta(\varphi, \psi, \pm\pi/2) = -A_\pm(\varphi \pm \psi)\sin\varphi + B_\pm(\varphi \pm \psi)\cos\varphi. \tag{3}$$

This situation is not very well suited for the construction of splines. But after transforming coordinates $\varphi = x + y$ and $\psi = x - y$ we can use the tensor product

$$s(x, y, \vartheta) = \sum_{ijk} c_{ijk} T_i(x) T_j(y) N_k(\vartheta)$$

of polynomial (N_k) and trigonometric (T_l) B-Splines to calculate an approximation s to a given function f. The coefficients c_{ijk} then are obtained as the solution of a large system of linear equations consisting of the peridocity and differentiability constraints and the normal equations.

The same approach can be used in a slightly modified form for approximation on S^3. In this case we will also consider a tensor product approach to the construction of a multiresolution setup and give a characterization of the corresponding wavelet spaces. By

introducing the usual quotient map from S^3 to SO(3) these function spaces can also be used for approximation on SO(3).

We will present an application of these methods to an approximation problem arising in molecular modelling and compare the results with other methods, for instance approximation by positive definite functions.

Compression through Rotations

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The use of approximation theory in the signal and image compression explores the appropriate choice of the basis in the respective functional space. The most successful in this direction are the wavelet bases, which are in some sense fractal structures and adapt themselves to the local peculiarities.

In this paper we define a rotation $U(\lambda)$; $|\lambda| < 1$ in the Hilbert space L_2 with an unconditional basis $\Phi = \{\varphi_i(x)\}_{i=0}^{\infty}$. Let $H_{k,p} \subset H_{0,0} = L_2$ be the span over $\{\varphi_{2^k i+p(x)}\}_{i=0}^{\infty}$; $k = 0, 1, 2, \ldots, p = 0, 1, 2, \ldots, 2^k - 1$. The rotation $U(\lambda)$ has the property that for a given $f \in H$, $f(x) = \sum_{i=0}^{\infty} \varphi_i(f)\varphi_i(x)$, there exists a $\lambda_0 \in (-1, 1)$, such that for the rotated basis $U(\lambda)\Phi = \{\varphi_i(\lambda; x)\}_{i=0}^{\infty}$ one of the values $A(f;\lambda) = \sum_{i=0}^{\infty}(\varphi_{2i}(\lambda; f))^2$ and $B(f;\lambda) = \sum_{i=0}^{\infty}(\varphi_{2i+1}(\lambda; f))^2$, $\left(A(f;\lambda) + B(f;\lambda) = \|f\|^2\right)$ is maximal and the other is minimal. By consecutive rotations of the spaces $H_{p,k}$ we obtain a representation of the function f which is appropriate for compression.

This approach explores the fundamental idea of the wavelet bases and the potential of the fractal transforms for compression.

Computer experiments of signal and image compression are given.

On the Unconditional Convergence of the Multivariate D^m-Splines

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One way for defining the multivariate splines is the variational approach which leads to the D^m-splines of Atteia:

$$s = s(f, m, \Delta, \Omega) = \operatorname{argmin}\left\{ \|D^m g\|_2 : g \in W_2^m(\Omega), g\big|_{\Delta} = f\big|_{\Delta} \right\}.$$

The D^m-splines inherit a lot of remarkable properties of the univariate polynomial splines of degree $2m-1$. In particular, for $f \in W_2^m$ the D^m-spline interpolation process converges unconditionally with respect to the mesh structure. In Sobolev spaces W_p^l different from

W_2^m, convergence of the D^m-splines can be provided on the meshes with bounded global mesh ratio, but if no restrictions are assumed, then divergence examples (in some W_p^l) already exist.

Here we study the problem of finding the necessary and sufficient conditions on l and p providing unconditional convergence of D^m-splines in W_p^l. This problem originated from Carl de Boor's conjecture that unconditional convergence of univariate interpolating splines of degree $2m-1$ takes place for any $f \in W_p^m$, where $1 \le p \le \infty$. This conjecture was verified for $m = 1, 2, 3$, and recently we had shown that it is true for any m whenever p is sufficiently close to 2.

We show that in contrast to the univariate case, the unconditional convergence of the multivariate D^m-splines in W_p^m takes place if and only if $p = 2$.

Adaptive Kernel Estimation of the Spectral Density with Boundary Kernel Analysis

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A data-adaptive non-parametric procedure for estimating the log-spectral density, $\theta(f) \equiv \ln[S(f)]$, is proposed. Our scheme has the following steps:

0) Compute the Fourier transform, $y(f)$ on a grid of size $2N+2$ and $\hat{\theta}_1(f) = \ln[|y(f+\Delta) - y(f-\Delta)|^2/2(N+1)]$ on a grid of size $N+1$.

1a) Kernel smooth $\hat{\theta}_1(f)$ with a kernel of order (0,4) for a number of different bandwidths, h_ℓ, and evaluate the average square residual (ASR) as a function of h_ℓ

$$ASR(h_\ell) = \sum_{n=1}^{N} |\hat{\theta}(f_n) - \hat{\theta}(f_n|h_\ell)|^2$$

where $\hat{\theta}(f_n|h_\ell)$ is the kernel estimate of $\theta(f)$ using bandwidth h_ℓ.

1b) Estimate the optimal $(0,4)$ global halfwidth using a goodness of fit method. Relate this to the optimal $(2,4)$ using the halfwidth quotient relation.

2) Evaluate the multitaper estimate of $S(f)$ on a grid of size $2N+2$. If the computational effort is not important, set $K = N^{8/15}$; otherwise choose K according to your computational budget.

3) Estimate $\theta''(f)$ by smoothing the multitaper estimate with global halfwidth $h_{2,4}$.

4) Estimate $\theta(f)$ by substituting $\hat{\theta}''(f)$ into the optimal halfwidth expression corresponding to the minimum of (14).

Relative convergence rates are given. Step 2 uses a family of orthonormal spectral windows followed by kernel smoothing. The multiple spectral window procedure presmoothes the empirical spectrum and reduces the unpleasant effects of the long tail of the log-chi square distribution. We use the sinusoidal tapers: $v_n^{(k)} = \sqrt{\frac{2}{N+1}} \sin(\frac{\pi k n}{N+1})$. These

tapers are an orthonormal family and their sensitivity to bias is asymptotically minimal. Multiwindowing reduces the expected mean square error by $(\frac{\pi^2}{4})^{.8}$ over simply smoothing the log tapered periodogram. The optimal number of tapers is $O(N^{8/15})$.

Near the spectral peaks and at $f = 0$ or $f = 1/2$ we modify the kernel shape to take into account the boundary. Kernels which minimize the expected mean square error are derived. These kernels are equivalent to using a linear weighting function in the local polynomial regression. It is shown that any kernel estimator that satisfies the moment conditions up to order m is equivalent to a local polynomial regression of order m with some non-negative weight function if and only if the kernel has at most m sign changes. Fast algorithms are proposed for computing the kernel estimate in the boundary region and for evaluating the Rice criterion.

To initialize a plug-in kernel estimator (Step 1b), we use the *fitted* average square residual criterion (ASR) i.e. we fit the to the parametric model: $ASR(h) = a_0/Nh + a_1 h^4$ where h is the kernel halfwidth. This parametric fit results in more accurate, less variable estimates of the starting halfwidth.

Jackson-Type Theorems on the Real Line

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The famous theorems of Jackson about the degree of approximation by trigonometric and algebraic polynomials have been object of many generalizations and applications. Typical for these results is that the approximated functions are bounded and so they allow the construction of suitable sequences of linear operators (so called Jackson operators), which lead to the desired estimates.

In our approach we deal with functions defined on the whole line, not necessarily bounded. In this case the classical approach does not work (a result of Albinus). We consider then a convenient metric and estimate the rate of convergence of a approximating sequence, when structural properties of the involved functions are known, e.g., the growth of their derivatives. The method may be applied, mutatis mutandis, to entire functions.

On Fan's Best Approximation and Applications

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Ky Fan proved the following theorem known as the best approximation theorem.

Let C be a compact convex subset of a Banach space X and $f : C \to X$ a continuous function. Then there is a $y \in C$ such that

$$\begin{aligned} \|y - fy\| &= d(fy, C), \\ &= \inf \{\|fy - x\| : x \in C\}. \end{aligned}$$

This theorem has applications in fixed point theory and variational inequalities. An extension of this theorem and applications will be presented in this talk.

On the Pointwise Convergence of Fourier Series with respect to Periodized Wavelets

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Let $\{V_j\}_{j=-\infty}^{\infty}$ be a multiresolution analysis with the father- and mother-wavelets respectively φ and ψ, satisfying condition (A): $|\varphi(\tau)|, |\psi(\tau)| \le \nu(|\tau|)$, where ν is a monotone decreasing summable on $[0, +\infty)$ function. The tensor product of m analyses $\{V_j\}_{j=-\infty}^{\infty}$ is an m-dimensional multiresolution analysis $\{\mathbf{V}_j\}_{j=-\infty}^{\infty}$ in $L_2(\mathbf{R}^m)$. The periodized wavelet basis of $\{\mathbf{V}_j\}_{j=-\infty}^{\infty}$ is an orthonormal basis in $L_2([0,1]^m)$. The Fourier series of functions $f \in L([0,1]^m)$ with respect to this basis are considered (we will call them wavelet Fourier series). We study the convergence at the Lebesgue points and at the strong Lebesgue points, i.e. points x such that

$$\lim_{d(I)\to 0} \frac{1}{\mu I} \int_I |f(x+t) - f(x)|\, dt = 0, \quad \sup_I \frac{1}{\mu I} \int_I |f(x+t) - f(x)|\, dt < \infty,$$

here I is an m-dimensional interval, symmetric respectively to the origin, $d(I)$ is the diameter of I, μI is the measure of I. It is proved that only under condition (A) the wavelet Fourier series of $f \in L([0,1]^m)$ converges at each strong Lebesgue point. In particular, convergence holds almost everywhere for $f \in L \log L([0,1]^m)$. If instead of (A) we assume condition (B): $|\varphi(\tau)|, |\psi(\tau)| \le \nu(|\tau|)$, where ν is a monotone decreasing on $[0, +\infty)$ function such that $\int_0^{\infty} \tau^{m-1} \nu_(\tau)\, d\tau < \infty$, then the wavelet Fourier series of $f \in L([0,1]^m)$ converges at each Lebesgue point. In particular, convergence holds almost everywhere. Condition

(B) is close to sharp because we have an example of multiresolution analysis with father-wavelet $\varphi(\tau)$ decaying as τ^{-m} such that there exists a function $f \in L([0,1]^m)$ for which the wavelet Fourier series diverges at some Lebesgue point.

Approximation on Compact Sets by RBFs

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This paper examines both the theoretical and practical aspects of using RBFs as approximants over compact sets in R^s. In particular, we point out the importance of choosing the centers and the inherent difficulties therein. Several real-world problems are presented to emphasize the importance of choosing the centers correctly.

Multivariate Periodic Interpolation and Wavelets

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The aim of the talk is to describe wavelet decompositions for periodic functions in $L_2(\mathbf{T}^d)$. Therefore we consider nested spaces of multivariate periodic functions forming a non-stationary multiresolution analysis.

The scaling functions of these spaces are defined as fundamental interpolants on a regular grid or a sparse grid, respectively. The corresponding wavelets and wavelet spaces are constructed. Using the interpolatory conditions we obtain the decomposition and reconstruction relations.

As an example we use certain de la Vallée Poussin means.

Our investigations generalize the results for the univariate case by J. Prestin, E. Quak, and K. Selig (1993/94) which were based on the paper by A. A. Privalov (1991) on orthogonal bases of the space $C_{2\pi}$.

An Algorithm for Minimal Norm Solutions of Linear Inequalities

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Consider a system of m real linear inequalities in n real variables. When this system is feasible, that is when this system is consistent, it is natural to single out an n-vector x of

least norm satisfying these inequalities. In this paper a method is given which accomplishes this task for the problem $Ax \geq b, \|x\|$ (min), where the norm on $\mathbf{R}^n$ is the ℓ^p-norm, $1 < p < \infty$. We show how this problem can be converted to a non-negative least distance problem: $j \geq 0, \|c - Eu\|_q$(min), where q as usual is the index conjugate to p, viz. $p+q = pq$. Algorithms we developed in some earlier papers are now applicable to this least distance problem. If r denotes the ℓ^q residual of this problem, then one can show that the case $r = 0$ corresponds to the infeasibility of the system of inequalities. On the other hand, if $r \neq 0$, then r' is p-dual as defined in our earlier papers can be computed. From this it is a small step to the solution of our present problem.

Asymptotic Distribution of Poles and Zeros of Best Rational Approximants to $|x|^\alpha$ on $[-1, 1]$

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Like in the polynomial case so also in rational approximation there is a fundamental interest in the approximants of the functions $|x|^\alpha, \alpha > 0$, on $[-1, 1]$. Recently the strong error formula

$$E_{nn}\left(|x|^\alpha, [-1,1]\right) = 4^{1+\alpha/2} \left|\sin \frac{\pi}{2}\alpha\right| e^{-\pi\sqrt{\alpha n}}$$

has been proved for the minimalrational approximation error

$$E_{nn}\left(|s|^\alpha, [-1,1]\right) := \inf_{r \in \mathcal{R}_{nn}} \| |x|^\alpha - r \|_{[-1,1]},$$

where $\|\dot{\|}_{[-1,1]}$ denotes the uniform norm on $[-1, 1]$.

Let r_n^* be the best rational approximant of degree n. It is shown that all poles and zeros of r_n^* lie on the imaginary axis, further poles and zeros interlace. As a consequence of this configuration we have overconvergence throughout both the right and left halfplane.

In the talk we shall investigate the asymptotic distribution of these poles and zeros. In addition the asymptotic distribution of extreme points of the error function on $[-1, 1]$ will be studied. These questions are important for an understanding of the specific approximation speed.

Typically, asymptotic formulae can be proved in two degrees of precision: The more precise formulae allow to determine the location of individual objects (poles, zeros, or extreme points) as $n \to \infty$. The less precise version gives a description only in terms of asymptotic densities.

In the investigation rational interpolation, weighted orthogonal polynomials, potential theory, and some results from the theory of elliptical integrals play a major role.

Circular Bernstein-Bézier Curves and Degree Raising

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The theory of circular Bernstein-Bézier polynomials was discussed in a recent paper of Alfeld, Neamtu, and Schumaker. Given coefficients c_0, c_1, c_2, ..., c_n, the nth degree circular Bernstein-Bézier polynomial is given by

$$p(\theta) = \sum_{i=0}^{n} c_i \binom{n}{i} \left(\frac{\sin\frac{1}{2}(\theta_2 - \theta)}{\sin\frac{1}{2}(\theta_2 - \theta_1)}\right)^{n-i} \left(\frac{\sin\frac{1}{2}(\theta - \theta_1)}{\sin\frac{1}{2}(\theta_2 - \theta_1)}\right)^{i} \qquad \theta \in [\theta_1, \theta_2]$$

The authors present a degree raising formula for circular Bernstein-Bézier curves, but unlike the standard case, the control polygon of a circular Bernstein-Bézier curve does not converge to the curve under degree raising. We show that with proper normalization, the control polygon does converge to the curve as $n \to \infty$.

On Multivariate Attenuation Factors

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We introduce the method of discrete attenuation factors for the approximate computation of multivariate discrete Fourier transforms.

We consider attenuation factors related with multivariate discrete Bernoulli functions and deduce a best approximation property of the corresponding method of attenuation factors.

Attenuation factors for the computation of multivariate Fourier coefficients were examined in [3]. Choosing a unique approach to the discrete and non-discrete settings based on the Fourier analysis of locally compact abelian groups, we emphasize the close relation between both cases and interpret results in literature from a more general point of view. Especially, we generalize the considerations in [1] and [2].

[1] M. G. Ber, W. N. Malozyomov, The best formulae for the approximate computation of discrete Fourier transforms, Comput. Math Math. Phys. **32** (12) (1992), 1533–1544.

[2] H. Ehlich, Untersuchungen zur numerischen Fourieranalyse, Math. Z. **91** (1966), 380–420.

[3] M. H. Gutknecht, Attenuation factors in multivariate approximation, Numer. Math. **51** (1987), 615–629.

Wavelet Analysis of Scattered Data on the Real Line

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This is joint work with C. K. Chui, K. Jetter, and J. D. Ward.

We provide a general framework for the construction of wavelets that are generated by translates of a given slowly growing function h where the translates are defined by a sequence of arbitrarily spaced points. Our approach is operator theoretic in nature and relies only on specific properties of the function h (e.g., the order of the singularity of its Fourier transform at the origin) and not on a detailed knowledge of the function itself.

The methods used here provide an exact identification of the L_2-spaces generated by linear combinations of shifts of h, and they allow the description of Riesz bases by certain coefficient sequences in $\ell_2(\mathbf{Z})$. Furthermore, assuming boundedness of the global mesh ratio of the sequence of points, the orthogonal decomposition of two embedded L_2-spaces is presented, and Riesz bases for the corresponding wavelet space are constructed.

Our investigation covers certain classes of functions, including splines with simple knots and the Hardy multiquadric. We also present an example of a regularized Haar wavelet for scattered data.

Strictly Positive Definite Functions on the Unit Sphere of the Complex Hilbert Space

Dedicated to E. Ward Cheney in Honor of His 65th Birthday

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Springfield, Missouri

Let S^∞ denote the unit sphere in the complex Hilbert space H. A continuous f defined on the unit disc $D := \{z : |z| \leq 1\}$ is said to be positive definite on S^∞ if for every natural number n and any n points $z_1, \ldots, z_n$ on S^∞, the matrix A with entries $A_{ij} = f(\langle z_i, z_j \rangle)$ is nonnegative definite, where $\langle \cdot, \cdot \rangle$ denotes the inner product on H. If the matrix A is positive definite whenever the n points $z_1, \ldots, z_n$ are distinct, then f is said to be strictly positive definite on S^∞. In this paper, we examine several sufficient and/or necessary conditions that indicate if a given function f is strictly positive definite.

Malvar Wavelets on Hexagons

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Malvar wavelets were originally developed from the lapped orthogonal transforms to eliminate the blocking effects in transform coding, which can be thought of as a family of local sinusoid bases. A general description of window functions for Malvar wavelets was given by Coifman and Meyer. With this description of window functions, Malvar wavelets were generalized to more general forms, for example, continuous-time case and discrete-time case. Most recently, a family of two-dimensional nonseparable Malvar wavelets on rectangular regions were obtained by Xia and Suter. These two-dimensional nonseparable Malvar wavelets have potential applications in many fields, including the simulation of turbulence flow. In addition, a discrete implementation of spatial-varying filter banks was also obtained.

In this work, we want to construct orthonormal bases or Malvar wavelets defined on a hexagon A, using local bases defined on its three rhombuses $A_j, j = 1, 2, 3$. In the following, small bold English letters, such as $\mathbf{x}, \mathbf{y}$, always denote two-dimensional vectors in $\mathbf{R}^2$. Let $f_{j,k}(\mathbf{x}), k \in \mathbf{Z}$, be an orthonormal basis for $L^2(A_j)$ for each $j = 1, 2, 3$, where $L^2(A)$ denotes all square integrable functions on A_j. A trivial way to construct an orthonormal basis for $L^2(A)$ is that we simply use the truncation windows $\chi_{A_j}(\mathbf{x})$, which is 1 for $\mathbf{x} \in A_j$ and 0 otherwise, then $f_{j,k}(\mathbf{x})\chi_{A_j}(\mathbf{x}), j = 1, 2, 3, k \in \mathbf{Z}$, form an orthonormal basis for $L^2(A)$. However, the basis elements in this construction may have discontinuities. This may cause long terms in signal expansion. Therefore, one desires other window functions $w_j(\mathbf{x})$ rather than the truncations $\chi_{A_j}(\mathbf{x})$ so that the windowed local bases form an orthonormal basis for $L^2(A)$ and moreover, they are continuous in A. To construct such window functions and bases is the main goal of this work. The method developed in this work can be easily generalized to Malvar wavelets defined on many hexagons.

A New Construction of Wavelets: The Lifting Trick

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and Katholieke Universiteit Leuven

In this talk we present a new construction of wavelets that are not necessarily translates or dilates of one particular function. The advantage of such wavelets is that they can be tailored to specific situations. For example, one can construct wavelets that are biorthogonal with respect to a weighted inner product, or wavelets that are defined on general subsets of a Euclidean space (such as curves or surfaces). Since the wavelets are not the translates and dilates of one function, the Fourier transform can no longer be used in their construction. We present a very simple, but quite powerful alternative, which we name the "lifting trick". This construction leads to wavelets that still have the powerful

properties of classical wavelets, such as biorthogonality, compact support, a fast transform, vanishing moments, interpolation, and smoothness. We will present one example, namely the construction of wavelets on a sphere, in greater detail.

Interpolating Wavelets on $[-1, 1]$ and the Sphere

Manfred Tasche
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The construction of wavelets on an interval was recently considered by Meyer 1992, Chui–Quak 1992, Cohen–Daubechies–Vial 1993 and some others. In the following we describe a new approach to polynomial wavelets on $[-1, 1]$ introduced by Kilgore–Prestin 1994. Here we apply Chebyshev transforms, related shifts (see Butzer–Stens 1977) and discrete cosine transforms. It is essential that we do not use interior and edge scaling functions and wavelets.

Our sampling space V_j ($j \in \mathbf{N}_0$) is spanned by Chebyshev-shifts of a Lagrange function φ_j with respect to the Chebyshev nodes $\cos(2^{-j}k\pi)$ ($k = 0, \ldots, 2^j$). Note that V_j coincides with the set of all polynomials of degree $\leq 2^j$. The chain of all V_j ($j \in \mathbf{N}_0$) forms a multiresolution of the weighted Hilbert space $L^2_w[-1, 1]$ (with $w(x) := (1 - x^2)^{-1/2}$ ($x \in (-1, 1)$). The wavelet space W_j ($j \in \mathbf{N}_0$) is the orthogonal complement of V_j in V_{j+1} and is spanned by Chebyshev-shifts of the wavelet ψ_{j+1}. The shift-invariant spaces V_j and W_j can be easily characterized by Chebyshev transforms and discrete cosine transforms.

Using tensor products of interpolating trigonometric wavelets and polynomial wavelets introduced above, we obtain new wavelets on the sphere. The scaling functions of level j ($j \in \mathbf{N}_0$) are chosen as Lagrange fundamental polynomials with respect to grid $(2^{-j}k\pi, \cos(2^{-j}l\pi))$ ($k, l = 0, \ldots, 2^j$). For the decomposition and reconstruction of a given function, we describe very efficient and numerically stable algorithms based on fast Fourier transform and discrete cosine transform. Numerical tests will be presented.

Complex Rational Uniform Approximation Applied to the Design of Iterative Methods for Linear Systems of Equations

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For solving the real nonsingular system of linear equations $x = Tx + c$, we consider stationary (k, l)-step methods defined by the iterative scheme

$$p_0 x_m = T\Big(\sum_{i=1}^{l} x_{m-i} q_i\Big) + c - \sum_{i=1}^{k} x_{m-j} p_j$$

where $p_0 \neq 0, \sum_{i=0}^{k} p_i = \sum_{i=1}^{l} q_i = 1$.

One way of determining the parameters p_i and q_j, is to find a rational approximation of the conformal mapping from the exterior of the unit disk onto the exterior of a region Ω that encloses the spectrum of T [1]. In the case of a polygonal domain Ω, Li [2] employed a least-square criterion for approximating the Schwarz-Christoffel mapping computed with the aid of SCPACK [3].

The present contribution is intended to provide some numerical comparison based on the use of uniform approximation procedures.

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Quaternions on Wavelet Problems

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There are several problems and applications where wavelets are used where inclusion of Quaternions could be of great help simplifying the formulation of the problem.

This paper shows first how Quaternion Wavelets can be built and some of its properties and later some applications first in its classical use of representing rotations and afterwards in not so classical applications dealing with high dimensions and providing a tool for additional steps of compression of information.

One of the big advantages of using Quaternions is their easy algebraic representation that fits specially for computational uses.

Data Fitting by Integer Quasi-Convex Functions

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Given n data points $f = (f_1, f_2, \ldots, f_n)$, the problem is to find a best quasi-convex fit $g = (g_1, g_2, \ldots, g_n)$ with integer g_i which minimizes a suitable norm $\|f - g\|$. The norms under consideration are least squares, ℓ_1 and uniform. Algorithms of polynomial

complexity (in n) are developed for computation of best fits. These problems arise in data-fitting and statistical estimation.

Divergence–Free Wavelet–Bases

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In the last few years the existence of divergence–free wavelets, that was shown by Battle and Federbush, was used for the proof of existence–theorems of classical solutions of the incompressible Navier–Stokes–Equations. For numerical purposes these vector fields have some disadvantages: they are in general not compactly supported and the computation of these wavelets seems to be difficult. In 1992 Lemarié-Rieusset constructed compactly supported divergence–free wavelets, whose computation was standard. His construction was based on a tensor-product approach, that seems not be helpful for applications in fluid dynamics.

In this talk we show the basic ideas of a new construction of vector–wavelets with vanishing divergence without using tensor products. It will be pointed out, that these wavelets give rise to a stable basis of the space of all divergence–free vector fields and that they are not hard to compute. We give some examples based on box–splines and show applications in the numerical treatment of incompressible Navier–Stokes–Equations.

Step-by-Step Algorithms in the Multidimensional Spline Approximation

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The multidimensional spline approximation, especially at the huge scattered mesh instead of regular grid, is the expensive computational procedure both in analytical and in finite element methods. In this communication we present an effective method for the solving of system for the variational FE analog of spline based on special tensor decomposition of multi-index matrices and vectors. It leads to non-expensive step-by-step improvement algorithm to provide an a priori given error bound. Simultaneously, the fluent linear increasing of data volume is provided in the steps, and effective data compression is achieved for the low accuracy requirements.

Construction of Fundamental Interpolants and Wavelets

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This is joint work with C. K. Chui. Our objective is to introduce a general scheme for the construction of interpolatory approximation formulas and compactly supported wavelets by using spline functions with arbitrary (nonuniform) knots. Both constructions use "optimally local" interpolatory fundamental spline functions which are not required to possess any approximation property.

Representing the Error in Multivariate Polynomial Interpolation

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I give a quick tour of my recent results concerning constructive instances of the error bounds that numerical analysts use to conclude that a scheme based on polynomial interpolation has the highest possible order of accuracy that its polynomial reproduction allows.

Highlights in the univariate case include the bound

$$|f(x) - H\Theta f(x)| \leq \frac{n^{1/q}}{n!} \frac{|\omega_\Theta(x)|}{\operatorname{diam}\{x,\Theta\}^{1/q}} \|D^n f\|_{L_q}(\operatorname{conv}\{x,\Theta\}),$$

where $H - \Theta f$ is the Hermite interpolant to f at the multiset Θ of n points, $\omega(x) := \prod_{\theta\in\Theta}(x-\theta)$, and **diam** (resp. **conv**) is the diameter (resp. convex hull) of a (multi)set of points. This inequality significantly improves upon *Beesack's inequality*, upon which almost all the error bounds for Hermite interpolation given over the last 30 years have been based. Some interesting pictures of the Birkhoff splines which describe the error in Hermite interpolation are presented.

Highlights in the multivariate case include the multivariate form of *Hardy's inequality*, that for $m - n/p > 0$

$$\| x \mapsto \int_{\underbrace{[x,\dots,x}_{m},\Theta]} f \|_{L_p(\Omega)} \leq \frac{\Gamma(m-n/p)}{\Gamma(m)\Gamma(\#\Theta + m - n/p)} \|f\|_{L_p(\Omega)}, \quad \forall f \in L_p(\Omega),$$

where Θ is a finite sequence of points in $\mathbb{R}^n$, and $f \mapsto \int_\Theta f$ is (up to a scalar multiple) the linear functional of integration against the simplex spline $M(\cdot|\Theta)$ (introduced by Micchelli, 1980). This inequality has great utility and can be applied to the many integral error formulæ that exist, including those for Kergin/Hakopian interpolation, and recent work of

Sauer and Xu. There is some discussion on formulæ for the error in linear interpolation on a triangle, including presentation of a formula which exhibits the geometry of the problem in a pleasing way.

Construction of Orthonormal Wavelets with Multiplicity r with the Same Support to Increase the Number of Vanishing Moments r-fold

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This is a joint work with C. K. Chui. Let $r \geq 2$ be any preassigned integer. To any compactly supported orthonormal scaling function whose integer-translates contain locally all polynomials of order m (or degree $m-1$) in their linear span, we construct $r-1$ additional scaling functions with the same support, so that the integer translates of all of the r scaling functions constitute an orthonormal family, and that together, they provide local approximation of order rm. In addition, we construct the r corresponding orthonormal wavelets with the same support. Consequently, these wavelets have rm (instead of just m) vanishing moments.

Cubic Spline-Wavelet Packets of Sobolev Spaces and Adaptive Approximation

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Let $H_0^2(I)$ denote homogenuous Sobolev space :

$$H_0^2(I) = \{f(x) \in H^2(I) |\, f(0) = f'(0) = f(L) = f'(L) = 0\},$$

with the inner product $< f, g >= \int_I f''(x)g''(x)\,dx$, where $I = [0, L]$. (For the sake of simplicity, we assume that L is an integer greater than 3.)

Let V_j be the subspace of all cubic splines in $H_0^2(I)$ with knots $\{\frac{k}{2^j}\}$. It is well-known that $\{V_j\}_{j=1}^{\infty}$ forms an MRA for $H_0^2(I)$ equipped with norm $|||f||| =< f, f >= \int_I f''(x)g''(x)\,dx$. Let f_j be the cubic spline in V_j interpolating f. Then f_j is the optimal approximation of f in V_j under the norm $|||\cdot|||$. The function f_j is uniquely determined by the vector $\mathbf{f^j} := \{f(\frac{k}{2^j})\}_{k=1}^{2^jL-1}$.

We construct two compactly supported cubic splines in V_1, $\psi(x)$ and $\psi_b(x)$ with $supp\psi = [0, 3]$ and $supp\psi_b = [0, 2]$. The j-level dilation and translations of ψ and j-level dilation and reflections of ψ_b form the basis of W_j, which is the orthogonal complement of V_j in V_{j+1}. Then $H_0^2(I) = V_0 \oplus_{j=1}^{\infty} W_j$. By the way, V_0 and $W_j, 0 \leq j < \infty$, also form a decomposition of the space C_0. Notice that any function g_j in W_j is uniquely determined

by $\mathbf{g^j} := \{g_j(\frac{2k-1}{2^{j+1}})\}_{k=1}^{2^jL}$ and $g_j(\frac{k}{2^j}) = 0$, $\forall k$. Because of this property, this kind of basis has two advantages. (1) It can be used to decompose sampling data directly. (2) Distinct from the usual wavelet basis in L^2, it provides an algorithm which is easy to perform by using parallel algorithm. That means, if the sampling data $\mathbf{f}_J := \{f(\frac{k}{2^J})\}_{k=1}^{2^jL-1}$ are given, we can decompose it into all levels in parallel.

Now we write $g_j(x) = \sum d_{j,k}\psi_{j,k}(x)$. The following theorem tells us that the coefficients of the decomposition can reflect the local singularity of the function.

Theorem 1. Let $0 < \alpha < 4$. Suppose the function f is Hölder continuous with exponent α at $x_0 \in I$. Then for any $k \in Z$, $j \in Z^+$ such that $2^{-j}k \in (x_0 - \delta/2, x_0 + \delta/2)$,

$$d_{j,k} = O(2^{-\alpha j}), \qquad j \to \infty.$$

Hence, for big j, only the coefficients corresponding to the singularities are relatively large. Then the terms with small coefficients can be deleted without causing an error beyond tolerance. The following theorem tells us that these terms can be essentially determined by the corresponding sampling values of g_j. Hence we can predetermine the terms to be deleted .

Theorem 2. Let $M = max_I|f(x)|$ and $g_j(x)$ be its wavelet component in W_j. Suppose that for $\epsilon > 0, -1 \le k_1 < k_2 \le n_j - 2$,

$$\left|g_j(\frac{2k-1}{2^{j+1}})\right| \le \epsilon, \qquad \text{for } k_1 \le k \le k_2.$$

Then the function

$$\tilde{g}_j(x) = \sum_{-1\le k\le n_j-2, k\ni[k_1+l,k_2-l]} d_{j,k}\psi_{j,k}(x)$$

satisfies

$$|\tilde{g}_j(x) - g_j(x)| \le C(M)\epsilon,$$

where $l = l(\epsilon) = [0.87 + \log\frac{1}{\epsilon}]$.

If the function f has only isolated singularities, the quantity of nonvanishing terms in each j-level are almost the same while the dimension of W_j is exponentially increasing. This fact is crucial for adaptive approximation, data compression, numerically solving PDE and other applications.

The coded data by using wave decomposition described above have poor frequency resolution when the level is high. In order to get finer frequency resolution in high level, we construct the spline-wavelet packet in Sobolev space $H_0^2(I)$ as following. Let $d_1, d_2, \cdots, d_n$ be a sequence of 0s and 1s, $\bar{d}_n = 1 - d_n$, and $\alpha_n = \sum_{i=1}^n d_i 2^{i-1}$. Define $W_s^{d_1d_2\cdots d_n} \subset W_s$ for $n > 0$ such that any $g_s^{d_1d_2\cdots d_n} \in W_s^{d_1d_2\cdots d_n}$ satisfies

$$g_s^{d_1d_2\cdots d_n}(\frac{k}{2^{n+s+1}}) = 0 \qquad k \in Z \setminus \{2^{n+1}Z + \alpha_n\}.$$

(We denote $W_s^{d_1 d_2 \cdots d_n} = W_s$, for $n = 0$.) Then $W_{s+1}^{d_1 d_2 \cdots d_n} = W_s^{d_1 d_2 \cdots d_n + 1} \oplus W_s^{d_1 d_2 \cdots \bar{d}_{n+1}}$ and $V_j = V_{j-s} \oplus_{l=1}^{s-1} \oplus_{(d_1 d_2 \cdots d_l)} W_{j-s}^{d_1 d_2 \cdots d_l}$. Each $W_s^{d_1 d_2 \cdots d_n}$, $n = 1, 2, \cdots$, has the same dimension $2^s L$.

Using the tensor product we can obtain the multidimensional spline-wavelet packet. For any function decomposed by such a packet, the nonvanishing coefficients of its components in the distinct subspace $W_s^{d_1 d_2 \cdots d_n} \otimes W_s^{d_1 d_2 \cdots d_k}$ have almost the same time (or space) locations in their corresponding coefficient matrices. The procedure to obtain the coefficients for the spline-wavelet packet basis is simpler than that for wavelet basis. It is particularly useful for multi-dimensional problems such as multivariable approximation, image processing and nonlinear PDE etc.

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Wavelets Associated with Periodic Basis Functions

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In this talk, we investigate a class of nonstationary, orthogonal, periodic scaling functions and wavelets generated by continuously differentiable periodic functions with positive Fourier coefficients; these functions, which were introduced long ago by Schoenberg, will be called *periodic basis functions* (PBFs).

The scaling functions and wavelets presented here have a number of attractive features. Their translates are orthonormal, thereby eliminating the need for dual functionals. Such wavelets also show greater stability, because, for orthonormal bases, the Riesz constants are unity. The algorithms for analyzing with them are ultimately simple. The decomposition and reconstruction coefficients can be computed in terms of the discrete Fourier transform, so that FFT methods apply for their evaluation. In addition, decomposition at the n^{th} level only involves 2 terms from the higher level, as opposed to the $2n$ terms one needed

with the earlier periodic wavelets. A similar result applies for reconstruction. Finally, for many PBFs, the corresponding scaling functions and wavelets have good localization properties. One of the main results in this paper is that both the scaling functions and wavelets derived from a large class of PBFs have "periodic" standard deviations that are $\mathcal{O}(1/n)$ at level n, where the notion of periodic standard deviation is that described by Breitenberger. In fact, a periodic "uncertainty principle" applies in this setting, and one can show that the angle/frequency uncertainty "window" is $\mathcal{O}(\sqrt{n})$ at level n, at least for scaling functions wavelets associated with a certain class of PBFs.

De-noising Using Wavelets and Cross Validation

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We present numerical results for noise reduction performed by modified wavelet reconstruction and address the automatic choice of the related threshold and shrinkage parameter without any prior knowledge about the noise variance. We show that the cross validation method can be a helpful tool for making this choice.

As an application, we consider the problem of removing noise from speech. Our model is $s(t_i) = x(t_i) + n(t_i)$ for uniform time samples, t_i, where $x(t_i)$ represents the clean speech to be estimated and the $n(t_i)$ are independent, identically distributed noise samples having zero mean and, possibly unknown, variance, σ^2. We investigate both thresholding and shrinkage of wavelet coefficients to perform this task with both prior and no prior knowledge of σ^2. Wavelet coefficients of s are nonlinearly driven toward zero based on their relation to a threshold. The estimate, $\tilde{x}$, reconstructed from these modified wavelet coefficients is supposed to approximate the underlying speech, x. Our investigations include the use of wavelets on the whole real line, periodized wavelets, and wavelets constructed on an interval, the latter two types being applied to windowed speech segments. We also consider orthogonal (spline and Daubechies' wavelets), semi-orthogonal (spline) wavelets, orthogonal trigonometric wavelets and wavelet packets.

With prior knowledge of σ^2, we choose a threshold parameter using the method of Donoho, based on Stein's unbiased estimate of risk.

When the variance is unknown, we estimate the optimal choice of threshold using methods of cross validation, ordinary and generalized. We demonstrate that this estimation is close to the ideal threshold parameter for test cases involving knowledge of the true speech signal.

Smooth Functional Subdivision Methods for Irregular Triangulations of the Plane

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For a wide class of subdivision methods over irregular triangulations, we derive necessary conditions for these schemes to produce C^k continuous limit surfaces. We next give sufficient conditions for these schemes to produce C^1 continuous limit surfaces. We conclude with an example of such an irregular C^1 subdivision method.

An Approximation Problem in the Analysis of Measurements

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At the 1992 Texas meeting, Zwick introduced a planar minimax algorithm for the analysis of measurements related to the acceptance or rejection of a manufactured object. The corresponding least squares problem can be solved as an orthogonal Procrustes problem, and the minimax problem can be interpreted as a weighted least squares problem. Algorithms based on this are considered, as well as related methods which apply to various extensions of the underlying mathematical problem.

Non-Stationary Multiresolution Analysis and Corresponding Biorthogonal Systems

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A generalization of the concept of Multiresolution Analysis is obtained by dropping the assumption that all spaces are generated by the dilates of a single function. A typical example arises in connection with exponential splines. Our objective is to construct a biorthogonal wavelet basis corresponding to exponential splines and to indicate possible applications. A central issue in this context is the efficient computation of the dual mask coefficients.

Fast Algorithms for Trigonometric Wavelet Packets and Applications in Speech Processing

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The aim of this talk is to describe wavelet packet functions and spaces for a trigonometric multiresolution analysis based on fundamental Lagrange interpolants. The corresponding algorithms for wavelet packet decomposition and reconstruction are investigated in detail. As an example, an application of trigonometric wavelet packets to speech processing is considered.

Characterization Theorem of Best L_p $(1 \leq p \leq +\infty)$ Approximation with Restricted Ranges of Derivatives

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We assume that $\Phi_n = \text{span}\{\phi_1, \phi_2, \ldots, \phi_n\}$ is a n-dimensional subspace of $L_p[a,b]$, $\phi_1^{(s)}, \phi_2^{(s)}, \ldots, \phi_n^{(s)}$ exist $(s = 0, 1, \ldots, k)$ and have a maximal linearly independent subset which is an extended Chebyshev system. Write

$$K_s = \left\{ q \in \Phi_n : l_s(x) \leq q^{(s)}(x) \leq u_s(x), x \in [a,b] \right\},$$

where l_s and u_s are extended real valued functions satisfying $-\infty \leq l_s(x) \leq u_s(x) \leq +\infty$. In this paper we give a characterization for uniform approximation to $f \in C[a,b]$ by $K_S : \cap_{s=1}^k K_s$. The result is widely applicable and containing many special cases such as approximation with HB interpolatory restrictions, comonotone approximation, and approximation by polynomials with bounded coefficients, etc. In case of $1 \leq p < +\infty$, we call $q = \sum_{j=1}^n a_j \phi_j$ a vector, define Euclidean inner product in Φ_n, and write $K_\Lambda = \{q \in \Phi_n : (q, h_\lambda) \leq d_\lambda, \lambda \in \Lambda\}$, where d_λ is a real number and h_λ a nonvanishing vector in Φ_n for each $\lambda \in \Lambda$. We say K_Λ is a "local convex cone" at $q_0 \in K_\Lambda$ if there exists a $\delta > 0$ such that $\{q \in \Phi_n : \|q - q_0\| < \delta\} \subset \{q \in \Phi_n : (q, h_\lambda) \leq d_\lambda, \lambda \in \Lambda \backslash \Lambda'\}$ with $\Lambda' := \{\lambda \in \Lambda : (q_0, h_\lambda) = d_\lambda\}$. Then we get a characterization theorem of L_p approximating from the product of K_S and a "local convex cone". This result is widely applicable as well.

Computing Continuous Wavelet Transforms By Subdivision

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In real applications, efficient algorithms are needed to compute wavelet transforms. For the continuous wavelet transform, Holschneider, Kronland-Martinet, Morlet and Tchamitchian proposed the efficient *algorithme à trous* which is based on a *hole filling process*. The hole filling process is limited in the sense that it uses spline interpolation that is restricted to piecewise constant and piecewise linear splines. In search of an efficient algorithm with higher order accuracy, we extend the *algorithme à trous* by just approximating the analyzing wavelet by a linear combination of refinable functions; approximating by quasi-interpolation in the spline case. The refinability permits subdivision schemes to be applied in the computation which lead to a very efficient new algorithm. The algorithm has an almost identical implementation as the *algorithme à trous* but the starting point is different. Our algorithm coincides with the *algorithme à trous* in the one dimensional case if the initial approximation is given by piecewise constant or piecewise linear spline interpolation. This coincidence reveals the connection between subdivision and hole filling, which are two basic approaches to render given data and provide approximation. The algorithm is an extension of *algorithme à trous* for it provides higher order approximation and easily generalizes to higher dimensions.

Fejér Means for Multiple Fourier Series

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This is joint work with Hubert Berens. The following result is an analogue of Fejér's theorem on Fejér kernel:

Theorem. *Let $f \in C(\mathbf{T}^d)$ and let $S_{n,d}^{(1)}(f)$ denote the n-th partial sum of its Fourier Series, where the sum is taken w. r. t. all multi-indices $\alpha \in \mathbf{Z}^d$ with modulus $|\alpha|_1 := |\alpha_1| + \ldots + |\alpha_d| \le n$, $n \in \mathbf{N}_0$. Then the Cesáro $(C, 2d-1)$ means of $S_{n,d}^{(1)}$ define a positive approximate identity; the order of summability is best possible.*

The proof of the theorem depends heavily on results from the theory of special functions. We also discuss the analogous results on Fourier integrals.

Construction of Wavelets on Finite Domains

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We will present the construction of wavelets on finite domains and their applications.

On Copositive Polynomial Approximation in $L_p[-1,1]$

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For function $f \in L_p[-1,1]$, $0 < p < \infty$, with finitely many sign changes, we construct a sequence of polynomials $P_n \in \Pi_n$ which are copositive with f and such that

$$\|f - P_n\|_p \le C\omega_\phi\left(f,(n+1)^{-1}\right)_p,$$

where $\omega_\phi(f,t)_p$ denotes the Ditzian-Totik modulus of continuity in L_p metric.

The Integral Representation of Bivariate Splines

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This is joint work with H. W. Liu. In this paper we give an integral representation of bivariate splines. As an application, we use such representation of splines to study the dimension problem of the C^1 quadratic spline space $S_2^1(\Delta)$ over some triangulation Δ.

Approximation in Norm and in Sobolev Seminorm

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For any ϕ in $W_2^m(R^s)$, the shift-invariant space $S(\phi)$ generated by ϕ provides simultaneous approximation order (m,k), with $k > m$, iff it provides approximation order $k-m$ in

the Sobolev seminorm $|\cdot|_{m,2}$. If each $S(D^\alpha\phi)$ provides approximation order k for $|\alpha| = m$ then $S(\phi)$ provides approximation order $k+m$.

On Signal Processing by Means of Periodic Spline Wavelets

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The objective of the presentation is an outline of techniques for adaptive signal processing (SP) based on the spline wavelet analysis.

Early examples of wavelets were based on spline functions. Later, spline wavelets were shadowed by the compactly supported orthonormal wavelets by I. Daubechies. However, in a good deal of situations spline wavelets offer advantages before these latter. Especially these manifest itself in SP, where the property of linear phase is required, which fails when employing the wavelets by I. Daubechies. In recent years cardinal spline-wavelets were subjected to detailed studying, mainly by C. K. Chui with collaborators. In particular, the authors succeeded in constructing compactly supported wavelets in the spaces of cardinal splines. The remarkable property of such spline wavelets, in addition to the symmetry, is that their dual wavelets belong to the same spaces as the original ones.

We discuss here periodic spline wavelets. Our approach to the spline wavelet analysis is based on an original computational technique—the so-called Spline Harmonic Analysis (SHA) which is a version of the Harmonic Analysis (HA) in spline spaces. SHA in some sense bridges the gap between the continuous and the discrete versions of HA. It is a rather universal technique applicable to a great variety of numerical problems, not necessarily to wavelet analysis.

The employment of SHA techniques in wavelet analysis is found to be remarkable fruitful. This approach has given a chance to construct a rich diversity of spline wavelets including periodizations of the above mentioned compactly supported spline wavelets and their duals as well as the selfdual wavelets by Battle and Lemarié. There constructed the so-called *high- and low-frequency mother wavelets* including locally supported (up to periodization) ones.

These mother wavelets generate a library of semi-orthogonal wavelet packet bases of spline spaces. Moreover, the SHA approach provides an efficient algorithm for changing from the B-spline basis to a wavelet and wavelet packet basis of the spline space and back.

These latter, together with some quadrature formulas established, enable us to suggest adaptive fast methods for digital processing of one- and multi-dimensional signals by means of spline wavelets and wavelet packets. We emphasize that the relations, being derived by means of SHA methods for periodic signals, provide algorithms for local processing non-periodic signals in real time conditions as well.

The algorithms suggested allow simple fast numerical implementation. There developed a software for SP on the basis of these ones.

A Good Interpolation Basis, the Marcinkiewicz-Zygmund Inequality and A_p Weights

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This is joint work with C. K. Chui. The equivalence of a good interpolation basis and the Marcinkiewicz-Zygmund inequality is established. It follows from the theory of singular integral operators and Carleson's interpolation theorem that they are equivalent to the separation of sample points and the A_p weight condition on the corresponding fundamental polynomial.

Asymptotic Expansions of the Representation Formulae for $(C_0)m$ Paremeter Operator Semigroups

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In this paper we expand asymptotically the general representation formulae for $(C_0)m$ parameter operator semigroups. When we consider special semigroups, our results yield the asymptotic expansions for multivariate Feller operators. In particular, the asymptotic expansions for univariate and multivariate Bernstein operators are reobtained. See the related examples at the end.

On the Approximation of Continuous Functions of Bounded Variation in the Space $C(\mathbf{R})$

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Let C be the space of continuous bounded functions, $f : \mathbf{R} \to \mathbf{R}$ with the norm $\|f\| = \sup_{x\in\mathbf{R}} |f(x)|$, V the set of functions $f : \mathbf{R} \to \mathbf{R}$ with the variation $\overset{\infty}{\underset{-\infty}{V}}(f) < \infty$, and $\omega(f,h) = \sup_{|t|\le h} \|f(\cdot + t) - f\|$ the modulus of continuity.

Theorem 1. *Let $A, \alpha, \beta > 0$, functions $f, g \in C \cap V$, $\int_{-\infty}^{\infty} dg = 1$. Then the series*

$$U_{\alpha,\beta}(f,g,x) := f(\infty)g(\infty) - f(-\infty)g(-\infty) - \sum_{k=-\infty}^{\infty} (f(\gamma_{k,\beta}) - f(\gamma_{k-1,\beta}))g((k\beta - x)/\alpha),$$

where $\gamma_{k,\beta}$ are arbitrary fixed points of the segment $[k\beta,(k+1)\beta]$ converges uniformly over $x \in \mathbf{R}$ and the inequality

$$\|f - U_{\alpha,\beta}(f,g)\| \le \omega(f,\alpha A) \underset{-A}{\overset{A}{V}}(g) + \omega(f,\beta) \underset{-\infty}{\overset{\infty}{V}}(g) + 2\|f\|\{\underset{-\infty}{\overset{A}{V}}(g) + \underset{A}{\overset{\infty}{V}}(g)\}$$

holds.

Corollary 1. *Let $A,\alpha,\beta > 0, f \in C \cap V$. Then*

$$R_{\alpha,\beta}(f) := \|f(x) - f(-\infty) - \sum_{k=-\infty}^{\infty} \{f(k\beta) - f((k-1)\beta)\} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(x-k\beta)/\alpha} e^{-u^2/2}\, du\|$$

$$\le \omega(f,\alpha A) + \omega(f,\beta) + \frac{4\|f\|}{\sqrt{2\pi}} \int_{A}^{\infty} e^{-t^2/2}\, dt.$$

If K is the class of functions $f \in C \cap V$ satisfying the conditions $\sup\limits_{f\in K} \|f\| < \infty$, $\lim\limits_{h\to 0+} \sup\limits_{f\in K} \omega(f,h) = 0$, then $\lim\limits_{\alpha,\beta\to 0+} \sup\limits_{f\in K} R_{\alpha,\beta}(f) = 0$. The theorem adduced develops in the several directions some results of V.I.Zubov (Dokl.Akad.Nauk SSSR. 316. (1991). N 6. 1298-1301) concerning to the approximation of probability distributions. The analogs of the theorem 1 and corollary 1 are also established for the case when the estimations are carried out by the means of the second modulus of continuity.

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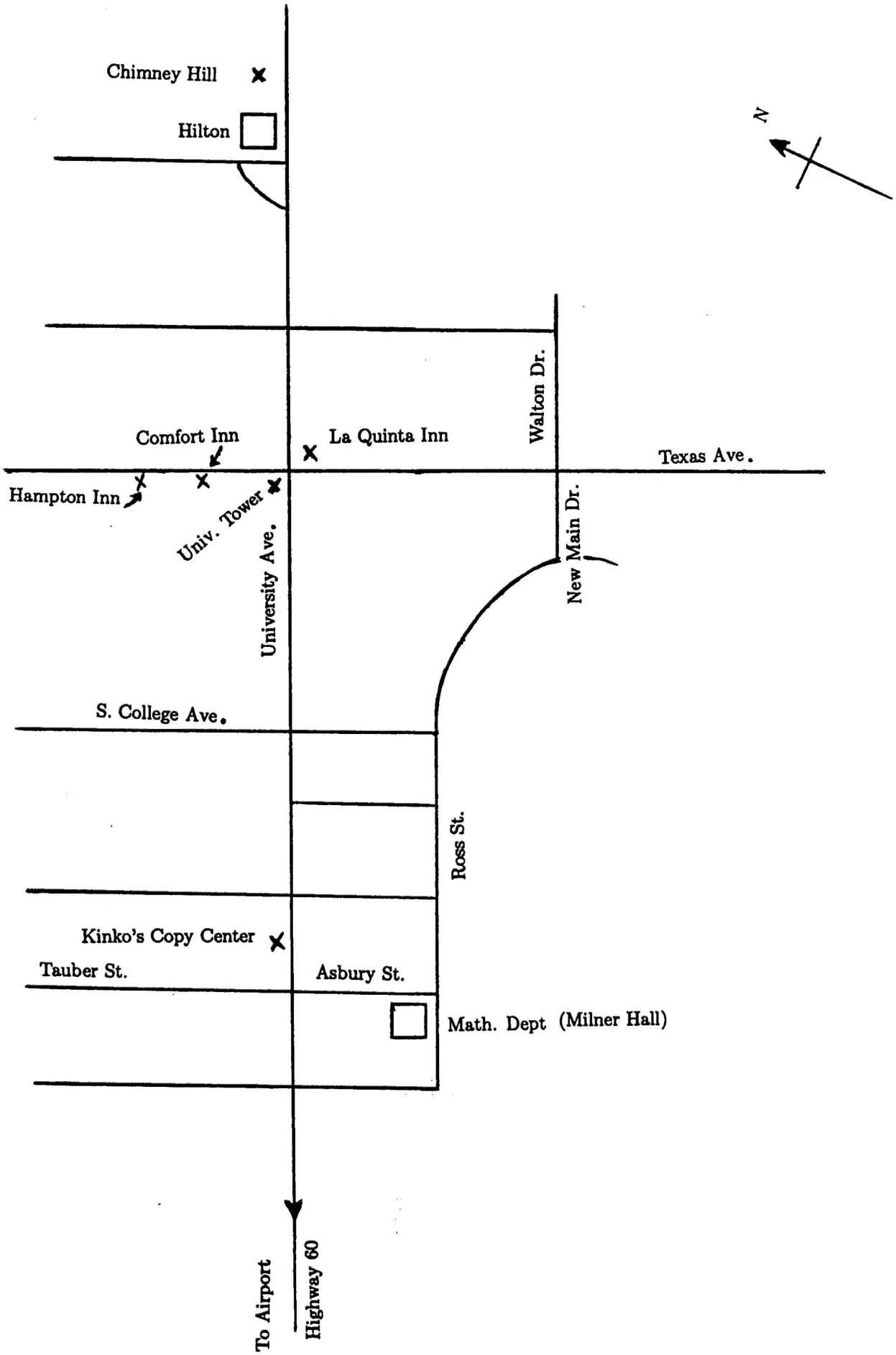
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Eighth International Conference on Approximation Theory – College Station, Texas, January 8-12, 1995

	Sunday, January 8		Monday, January 9		Tuesday, January 10		Wednesday, January 11		Thursday, January 12	
	Session 1A	*Session 1B*	*Session 6A*	*Session 6B*	*Session 11A*	*Session 11B*	*Session 16A*	*Session 16B*	*Session 21A*	*Session 21B*
8:20	Lenze	Blatter	Damelin	Chen, D	Boyadjiev	Sprengel	Petras	He, T X	Chen, G R	Lian, J A
8:40	Baxter	Prolla	Mthembu	Donovan	Levin	Tasche	Estrada	Hogan	Liu, X L	Petukhov
9:00	Beatson	Chalmers	He, M	Marasovich	Maimeskul	Prestin	Iske	Lodha	Stanley	Sendov
9:20	Fasshauer	Bacopoulos	Kochneff	Weinreich	Leeming	Weyrich	Sauer	Plonka	Anderson	Guerrini
9:40	Sun, X P	Singh	Fischer	Dahlke	Thiry	Quak	Levesley	Puccio	Muttiah	Opfer
10:00	Li, X	Peterburgsky	Petrushev	Hardin	Saff	Warhola	Baker	Kobayashi	Watson	Urban
10:20	**Break**		**Break**		**Break**		**Break**		**Break**	
	Session 2		*Session 7*		*Session 12*		*Session 17*		*Session 22*	
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12:00	**Lunch**		**Lunch**		**Lunch**		**Lunch**		**Lunch**	
	Session 3		*Session 8*		*Session 13*		*Session 18*		*Session 23*	
1:30	Micchelli		Popov Prize Winner		Lubinsky		Daubechies		Schaback	
	Session 4A	*Session 4B*	*Session 9A*	*Session 9B*	*Session 14A*	*Session 14B*	*Session 19A*	*Session 19B*	*Session 24A*	*Session 24B*
2:30	Le Méhauté	Suter	Stahl	Daubechies	Berens	Nashed	Natarajan	Jetter	Dyn	Oswald
2:50	Nürnberger	Wang, J Z	de Bruin	DeVore	Xu, Y	Jawerth	Foley	Stöckler	Reif	de Villiers
3:10	Lai, M J	Lorentz, R	Freund	Sweldens	von Golitschek	Houdre	Neamtu	Narcowich	Kobbelt	Montefusco
3:30	Hong, D	Lorentz, G	Driver	Aldroubi	Haußmann	Heil	Bajaj	Ward	Warren	Gomes
3:50	**Break**		**Break**		**Break**		**Break**		**Break**	
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5:30	Marano	Kopotun	Calvi	Maesumi	Shadrin	Felten	Santi	Pöplau	Bertram	Li, C
5:50	Davydov	Ubhaya	Lutterodt	Papadakis	Vasilenko	da Silva	Riedel	Waldron	Zhuk	Wang, J R
6:10	Dechevsky	Sreedharan	Ditzian	Traversoni	Kunkle	Polster	Sidorenko	Chen, T P	Xu, S S	Novikov, S
6:30	Demidovitch	Cao, J D	Nevai	Ron	Xu, Y S	Kilgore	Zhong, L F	Andrievskii	Melkman	Filippov
6:50			Badr	Novikov, I	Zhang, G	Pinter	Bokhari	Pai	Dong, X	

Ballroom V – Session A
Brazos Amphitheatre – Session B and one-hour addresses
Mockingbird Room AB – Discussions
Mockingbird Room CD – Book Displays